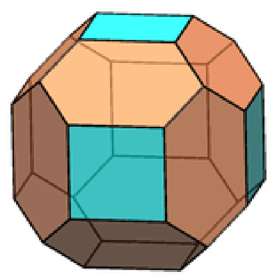
D.N.R.COLLEGE (AUTONOMOUS), BHIMAVARAM
I B.Sc., GEOLOGY (Crystallography)

Crystallography is a branch of mineralogy dealing with the science of crystals in their development and growth, their crystalline form and internal structure.

Crystals: crystals are solids bounded by flat surfaces arranged on a definite pattern which is an expression of internal atomic structure i.e. a homogeneous solid possess long-range, three-dimensional internal order (or) Crystal is a substance in which the constituent atoms, molecules, or ions are packed in a regularly ordered, repeating three-dimensional pattern.



The term crystal is derived from the Greek word krustallos for ice. It was applied originally to the water-clear quartz, or rock crystal, of the Alps, in the belief that this was really ice that had been subjected to such intense and long- continued cold that it could not melt.

Amorphous and Crystalline states: Crystal is formed by the solidification of minerals from the states of liquid or gas, is called crystallization.

When a mineral is incapable of developing of both external form and internal atomic structure is said to be **amorphous**. A mineral is said to be **massive**, when it shows no external structure. A very finely crystalline aggregate in which the crystals are so small as to be indistinguishable, except under microscope, is called **crypto crystalline.**

If the process of crystallization be slow, large crystals are built up, but the more rapidly it takes place the smaller are crystals, as there is no time for the molecules to gather together to form larger regular structures. If the transition to the solid state is very rapid, there is frequently no evidence of the existence of crystalline structure, and the substance is regarded as non-crystalline or amorphous.

Crystal aggregates are made up of two or more crystals. If the crystalline structure of the minerals varies from fibre to fibre and grain to grain. It is said to be **crystalline aggregate.**

Morphology of crystals:

Faces: crystals are bounded by smooth plane or surfaces called faces. Crystal faces are of two kinds, like faces and unlike faces. Faces which have the same properties are said to be like faces and faces that have different properties are called unlike faces.

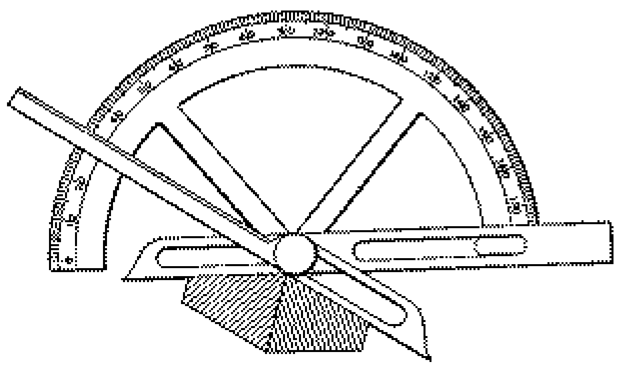
Edge: An edge is formed by the intersection of any two adjacent faces and such represents the intersection of two adjacent atom planes.

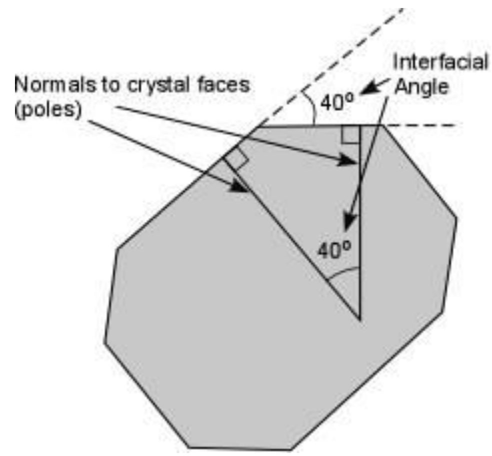
Solid angle or corner: It is a point where three or more faces meet or the intersection of three or more atom-planes.

Interfacial angle: The angle between any two adjacent faces of a crystal is termed as interfacial angle, but in crystallography the angle between the normals to the two adjacent faces is the "interfacial angle".

Measurement of interfacial angle:

The interfacial angles of crystals are measured by the Goniometer. Two types of this instruments are used, one termed the **Contact-goniometer**, the other the Reflecting goniometer.





The contact goniometer was introduced by France scientist Carangeot in 1780 .The contact-goniometer consists of two straight-edged arms movable on a pivot or screw, and connected by a graduated arc. These two arms are brought accurately into contact with adjacent faces of the crystal, and the angle between them read off on the graduated arc. In the illustration, the angle actually measured in the internal angle between the two faces, and this must be subtracted from 180^0 to give the interfacial angle used by the crystallographer.

Law of constancy of interfacial angle:

The atomic structure for the crystals of any one mineral is fixed, so that it follows that the positions of the faces of such crystals are fixed also. Therefore, angle between similar faces on all specimens of a particular mineral species will be essentially constant. This is the law of constancy of interfacial angles, a fundamental law of crystallography.

Euler's formula:

F+C = E+2 Where F=faces

C=corners (solid angles)

E=edges

Ex: cube: faces+corners = edges + 2

[6 + 8 = 12 + 2]

Zone: On both simple and multi-face crystal it is commonly apparent that the lines of intersection of several or many pairs of faces are said to constitute a zone. A line through the center of a crystal to which the edges of intersection of faces in a zone are parallel is called a zone axis.

Form: A form is said to constitute a group of faces generated from an initial face by the operation of the symmetry elements of a particular crystal class.

A crystal is bounded by all like faces are termed a simple form. Ex: cube, octahedron

A crystal which consists two or more simple forms is called a combination.

Ex: cube+octahedran

An open form is an assemblage of faces which cannot enclose a volume in space and can only occur in combinations. Ex: pinacoids

A closed form is an assemblage of faces which can enclose a volume in space. Ex: pyramid

Holohedral form: The form having all the faces which have the same position with regard to the crystallographic axes is called holohedral. Holohedral forms show the highest symmetry elements of the system. Ex: octahedron

Hemihedral form: The form having half the number of faces, that are present in the corresponding form of the normal symmetry class of the system, is called hemihedral. To one holohedral form there are two similar and complementary hemihedral forms called respectively positive and negative which together embrace all of its faces.

Ex: Tetrahedron is hemihedral form of octahedron.

Tetratohedral form: It has only one-fourth of the number of faces belonging to the normal form.

Hemimorphic form: In this form, faces develop only at the one end of a crystallographic axis. Hemimorphic form has no centre of symmetry.

Enantiomorphous forms: These forms occur in two positions, which are mirror images of each other. These forms have no plane of symmetry and no centre of symmetry.

Crystal form is named according to the number of its faces and their mutual relations.

Pedion: It has a single face. (Open form)

Pinacoid: It has a pair of parallel faces, on opposite sides of the centre. (Open form)

Prism: It has a set of equivalent faces, which are parallel to the vertical axis; and cut all the horizontal axes. (Open form)

Pyramid: It has a set of non-parallel faces, which are equally inclined to the vertical axis; and cut all the crystallographic axes. (Closed form)

Dipyramid: This can be considered as formed from two pyramids by reflection across a horizontal symmetry plane. (Closed form)

Dome: Two non-parallel faces symmetrical with respect to a symmetry plane. Faces are parallel to one horizontal axis only. (Open form)

Sphenoid: Two non-parallel faces symmetrical with respect to a 2-fold or 4-fold symmetry axis. (Open form)

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Symmetry

An object that consists of a systematic repetition of identical features is said to have symmetry.

Three main types of external symmetry are recognized:

(1) Symmetry with respect to plane – plane of symmetry
(2) Symmetry with respect to a line – axis of symmetry
(3) Symmetry with respect to a point- centre of symmetry

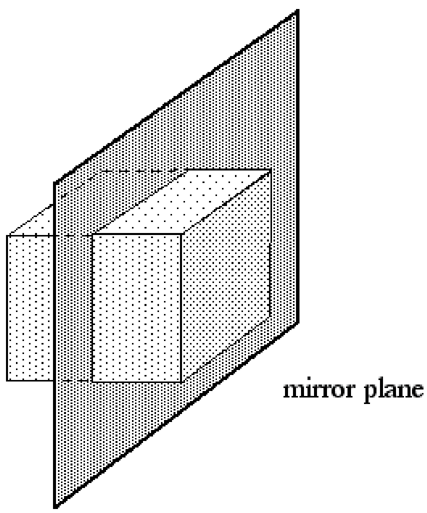
These are called symmetry elements.

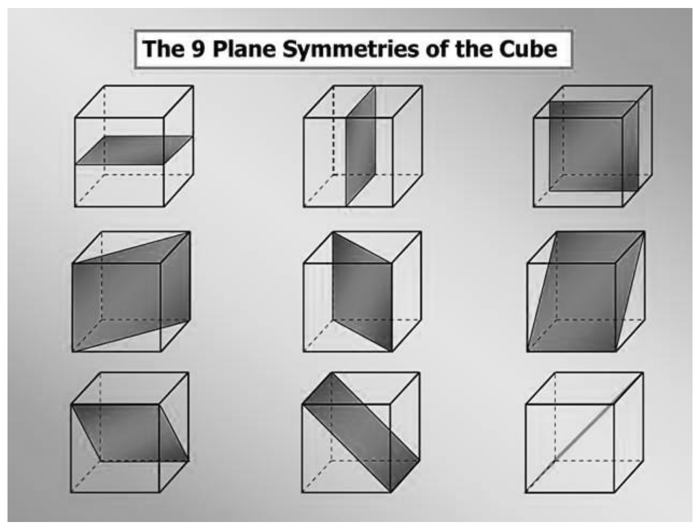
Symmetry is directly related to the atomic structure of crystals, the greater symmetry inside the crystal; the greater it is shown by balanced patterns of faces on the outside.

Plane of symmetry: A plane of symmetry divides a crystal into two similar and similarly-placed halves; such a plane divides the crystal so that one half is the mirror-image of the other.

Ex: cube has 9 planes of symmetry [3 axial and 6 diagonal]

Plane of Symmetry





Axis of symmetry: If a crystal, on being rotated, comes to occupy the same position in space more than once in a complete turn, the axis about which rotation has taken place is called an axis of symmetry. Depending upon the degree of symmetry, a crystal may come to occupy the same position two, three, four or six times in a complete rotation.

The terms are:

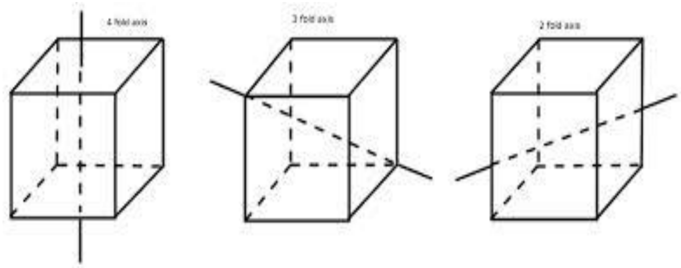
Two-times (ii): two-fold, diad, half-turn, diagonal axis, binary-180^0

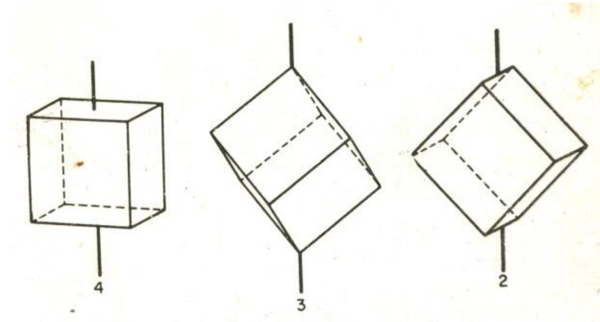
Three times (iii): three-fold, triad, one-third turn, trigonal axis-120^0

Four times (iv): four-fold, tetrad, quarter turn, tetragonal axis-90^0

Six times (vi): six-fold, hexad, one-sixth turn, hexagonal axis-60^0

Axis of Symmetry

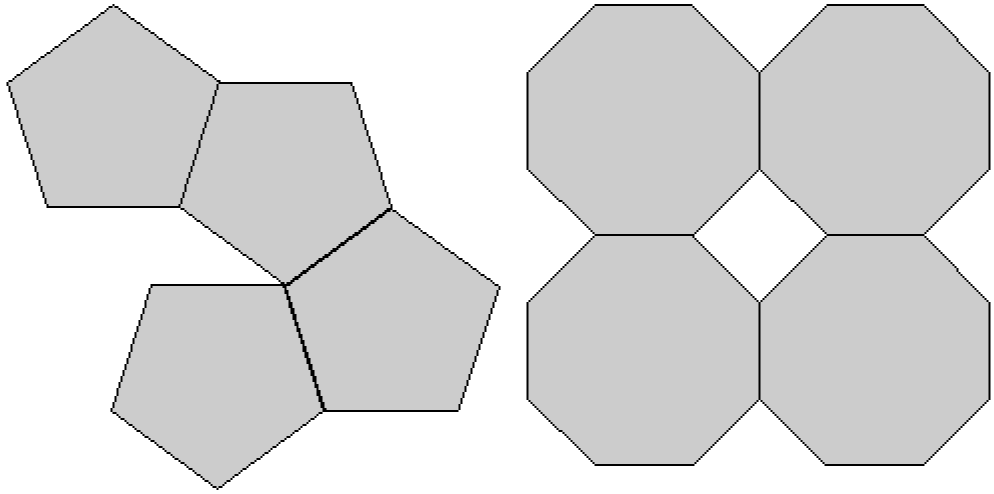




Five-fold symmetry is not possible (72^0turn will give non-integer values)

The reason is that the external shape of a crystal is based on a geometric arrangement of atoms. Note that if we try to combine objects with 5-fold and 8-fold apparent symmetry, that we cannot combine them in such a way that they completely fill space, as illustrated below.

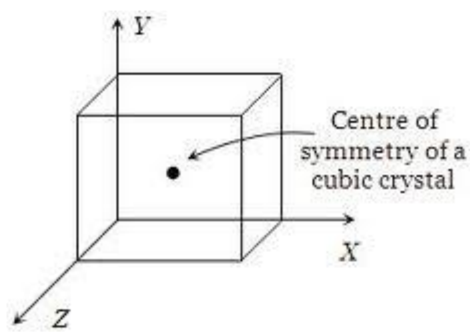
(It is interesting and important to remember that no crystal can have a 5-fold axis of symmetry. However, individual molecules *can* have such an axis of symmetry. A crystal cannot have a 5-fold axis because there is no way of packing by translation an infinite number of unit cells which have a pentagonal cross section so that they completely fill space.)



Centre of symmetry: A crystal has a centre of symmetry when like faces, edges.etc, are arranged in pairs in corresponding positions and on opposite sides of a centre point.

Ex: Cube has a centre of symmetry.

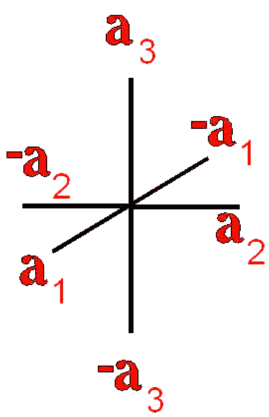
Centre of Symmetry



Composite symmetry: It is a composite element which combines rotation about an axis with inversion about the centre.

Crystallographic axes

One of the imaginary reference lines passing through the center ofan ideal crystal, designated **a, b, c**. or **a_1, a_2, a_3, c.**

<table>
<tr>
<td>Cubic System
$a = b = c$; $\alpha = \beta = \gamma = 90^0$

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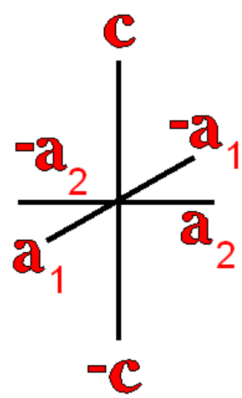
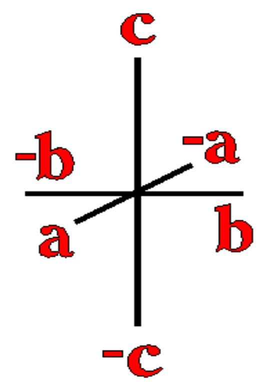
<td>Tetragonal System
$a = b \neq c$; $\alpha = \beta = \gamma = 90^0$

</td>
<td>Orthorhombic System
$a \neq b \neq c$; $\alpha = \beta = \gamma = 90^0$

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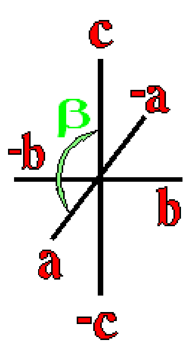
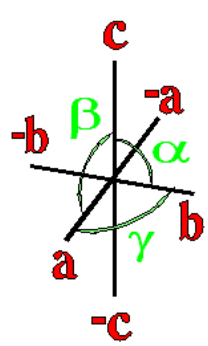
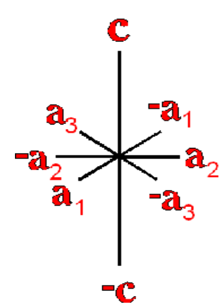
<tr>
<td>Monoclinic System
$a \neq b \neq c$; $\alpha = \gamma = 90^0$ $\beta \neq 90^0$

</td>
<td>Triclinic System
$a \neq b \neq c$; $\alpha \neq \beta \neq \gamma \neq 90^0$

</td>
<td>Hexagonal System
$a_1 = a_2 = a_3 \neq c$; (it has 4-axes)
$\beta_1 = \beta_2 = \beta_3 = 90^0$,
$\gamma_1 = \gamma_2 = \gamma_3 = 120^0$


Trigonal System
$a_1 = a_2 = a_3 = c$; (it has 4-axes)
$\beta_1 = \beta_2 = \beta_3 = 90^0$,
$\gamma_1 = \gamma_2 = \gamma_3 = 120^0$</td>
</tr>
</table>

<u>The 48 Special Crystal Forms</u>

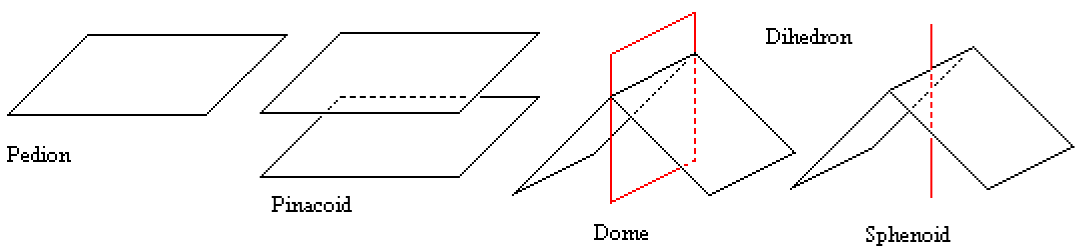
Forms, Open and Closed

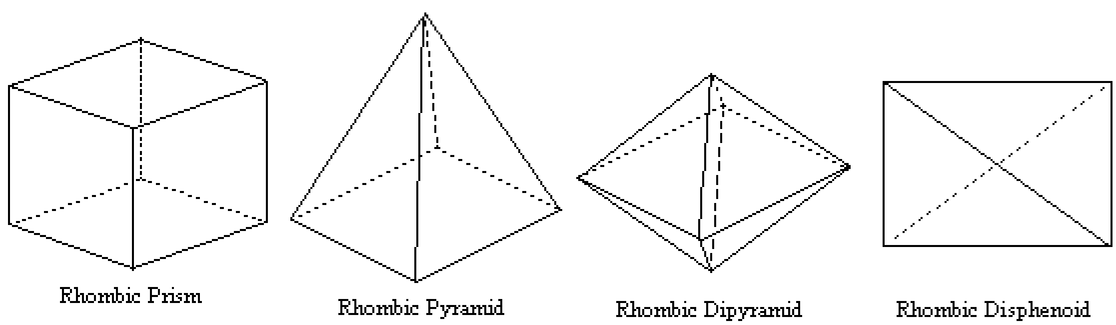
Any group of crystal faces related by the same symmetry is called a *form*. There are 47 or 48 crystal forms depending on the classification used.

Closed forms are those groups of faces all related by symmetry that completely enclose a volume of space. It is possible for a crystal to have entirely faces of one closed form. Open forms are those groups of faces all related by symmetry that do not completely enclose a volume of space. A crystal with open form faces requires additional faces as well. There are 17 or 18 open forms and 30 closed forms.

Triclinic, Monoclinic and Orthorhombic Forms

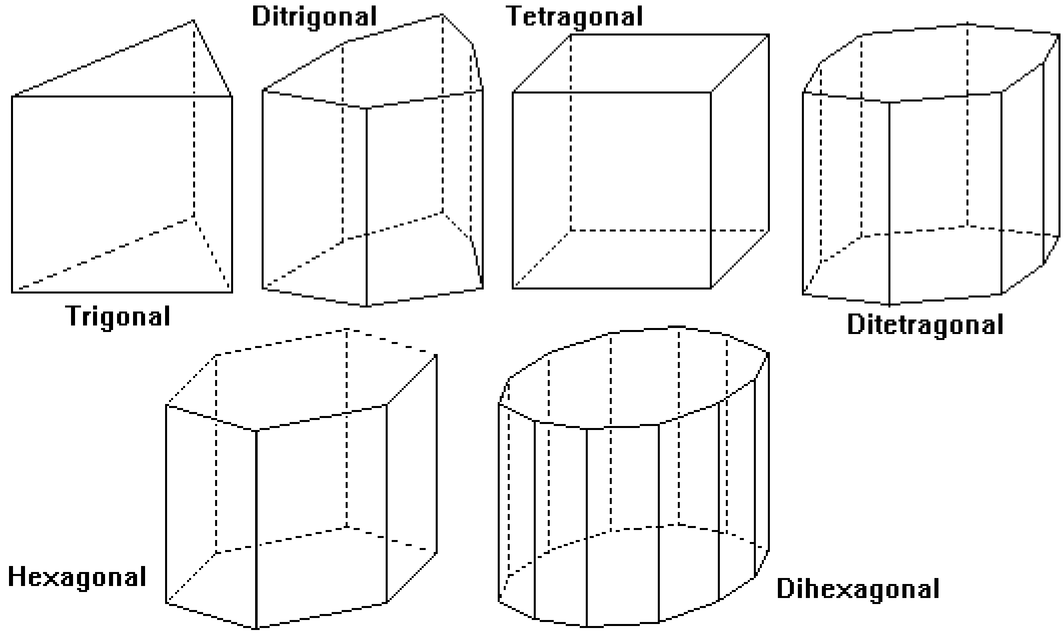
<u>Pedion:</u> A single face unrelated to any other by symmetry. Open
<u>Pinacoid:</u> A pair of parallel faces related by mirror plane or twofold symmetry axis. Open
<u>Dihedron:</u> A pair of intersecting faces related by mirror plane or twofold symmetry axis. Some crystallographers distinguish between domes (pairs of intersecting faces related by mirror plane) and sphenoids (pairs of intersecting faces related by twofold symmetry axis). All are open forms





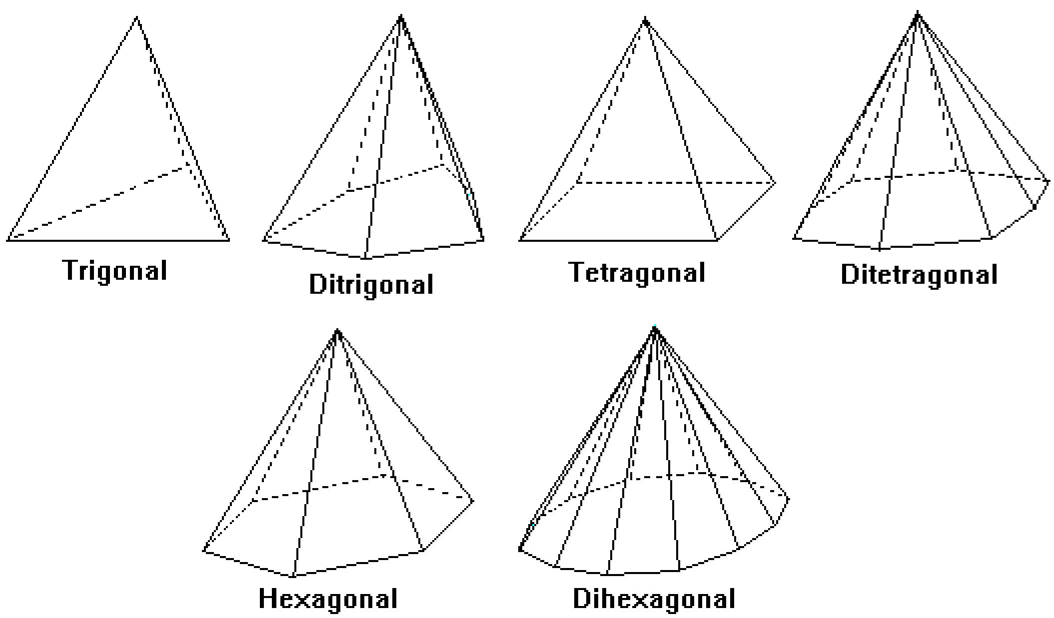
3-, 4- and 6-Fold Prisms

<u>Prism:</u> A collection of faces all parallel to a symmetry axis. All are open.



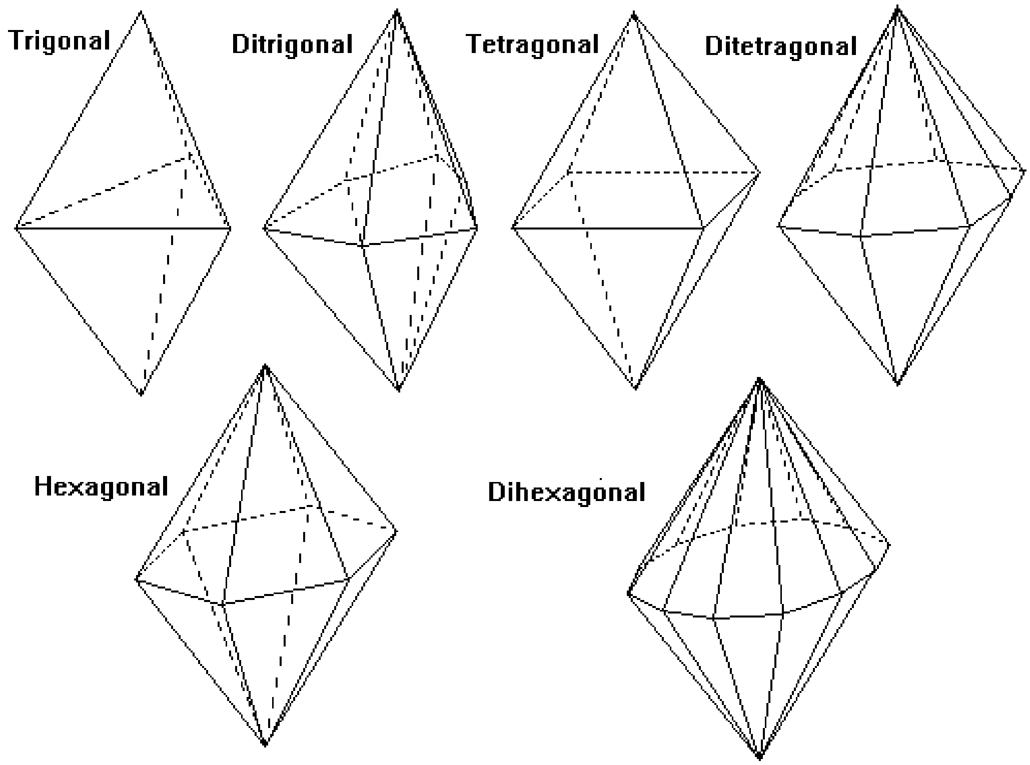
3-, 4- and 6-Fold Pyramids

Pyramid: A group of faces intersecting at a symmetry axis. All are open. The base of the pyramid would be a pedion.



3-, 4- and 6-Fold Dipyramids

Dipyramid: Two pyramids joined base to base along a mirror plane. All are closed, as are all following forms.



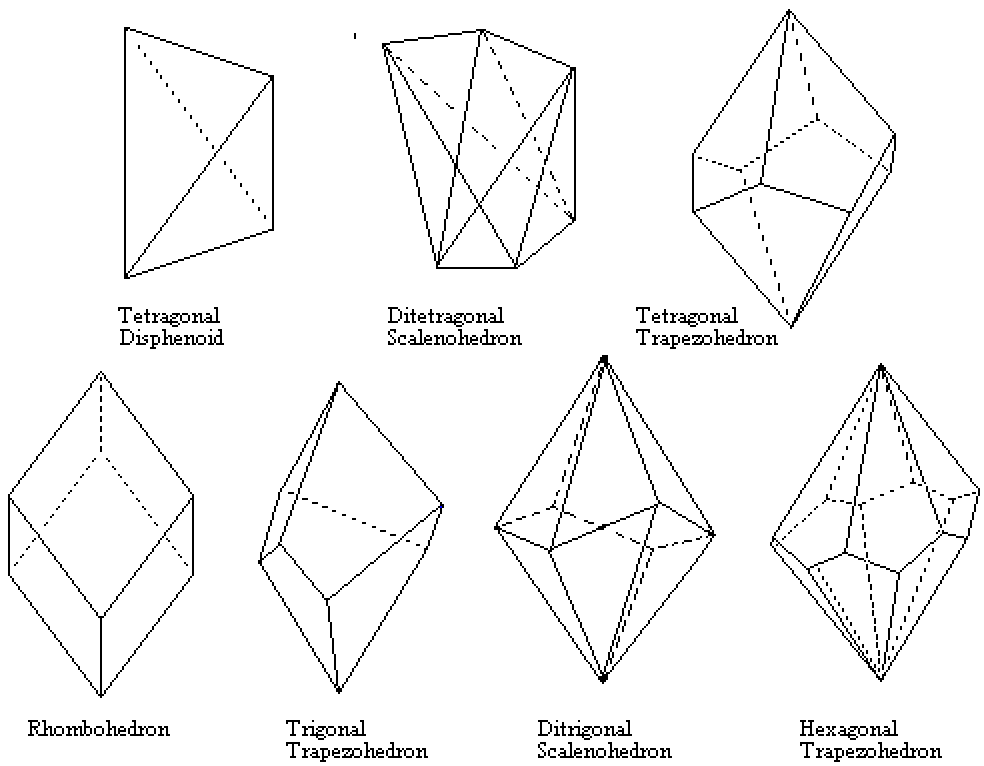
Scalenohedra and Trapezohedra

Disphenoid: A solid with four congruent triangle faces, like a distorted tetrahedron. Midpoints of edges are twofold symmetry axes. In the tetragonal disphenoid the faces are isoceles triangles and a fourfold inversion axis joins the midpoints of the bases of the isoceles triangles.

Scalenohedron: A solid made up of scalene triangle faces (all sides unequal)

Trapezohedron: A solid made of trapezia (irregular quadrilaterals)

Rhombohedron: A solid with six congruent parallelogram faces. Can be considered a cube distorted along one of its diagonal three-fold symmetry axes.



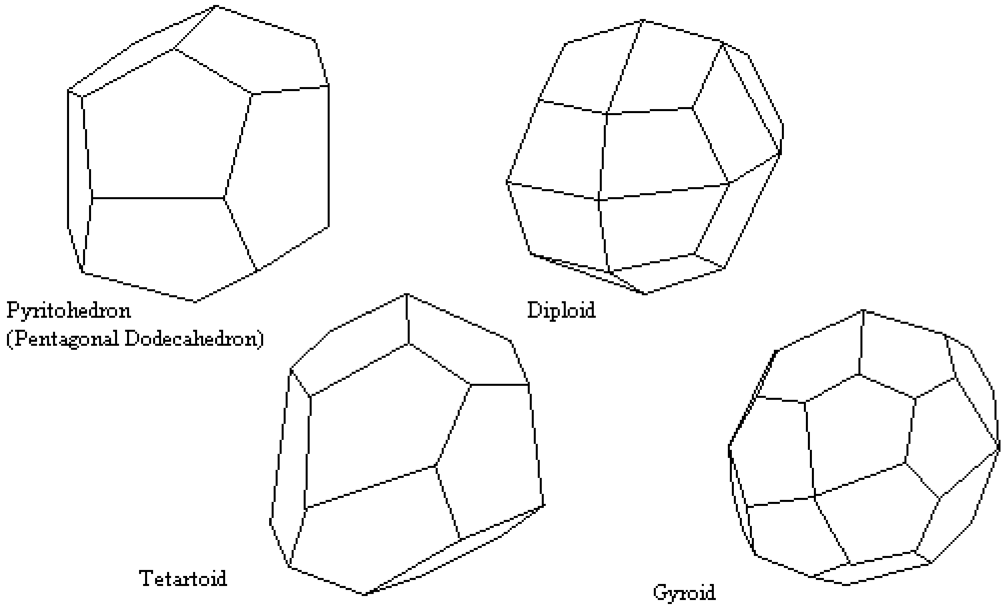
Tetartoidal, Gyroidal and Diploidal Forms

Tetartoid: The general form for symmetry class 233. 12 congruent irregular pentagonal faces. The name comes from a Greek root for one-fourth because only a quarter of the 48 faces for full isometric symmetry are present.

Gyroid: The general form for symmetry class 432. 24 congruent irregular pentagonal faces.

Diploid: The general form for symmetry class 2/m3*. 24 congruent irregular quadrilateral faces. The name comes from a Latin root for half, because half of the 48 faces for full isometric symmetry are present.

Pyritohedron: Special form (hk0) of symmetry class 2/m3*. Faces are each perpendicular to a mirror plane, reducing the number of faces to 12 pentagonal faces. Although this superficially looks like the Platonic solid with 12 regular pentagon faces, these faces are not regular.



Hextetrahedral Forms

<u>Tetrahedron:</u>

Four equilateral triangle faces (111)

<u>Trapezohedral Tristetrahedron:</u>

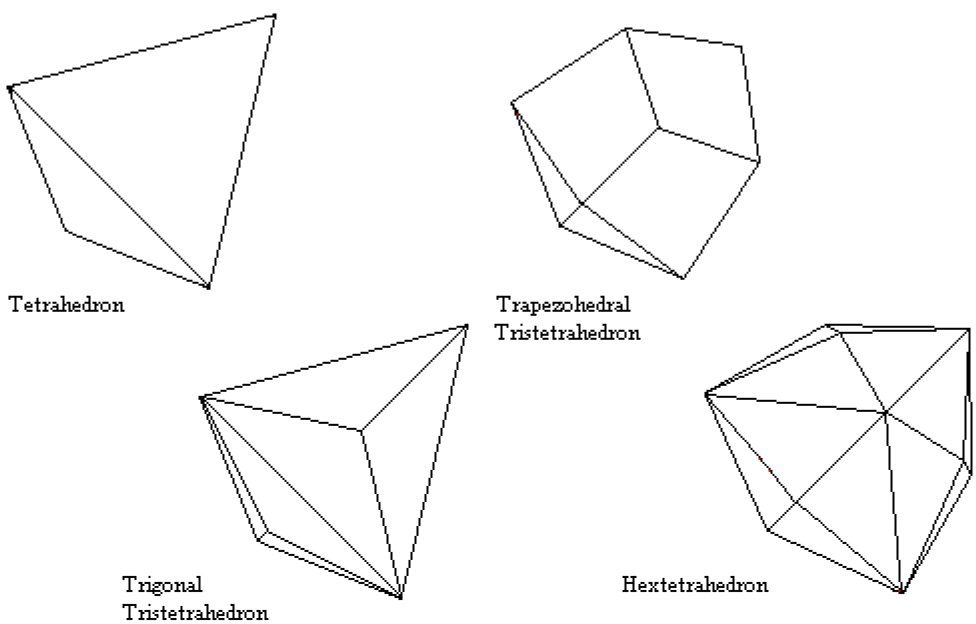
12 kite-shaped faces (hll)

<u>Trigonal Tristetrahedron:</u>

12 isoceles triangle faces (hhl). Like an tetrahedron with a low triangular pyramid built on each face.

<u>Hextetrahedron:</u>

24 triangular faces (hkl) The general form.



Hexoctahedral Forms

Cube:

Six square faces (100).

Octahedron:

Eight equilateral triangle faces (111)

Rhombic Dodecahedron:

12 rhombic faces (110)

Trapezohedral Trisoctahedron:

24 kite-shaped faces (hhl). Note that the Miller indices for the two trisoctahedra are the opposite of those for the tristetrahedra.

Trigonal Trisoctahedron:

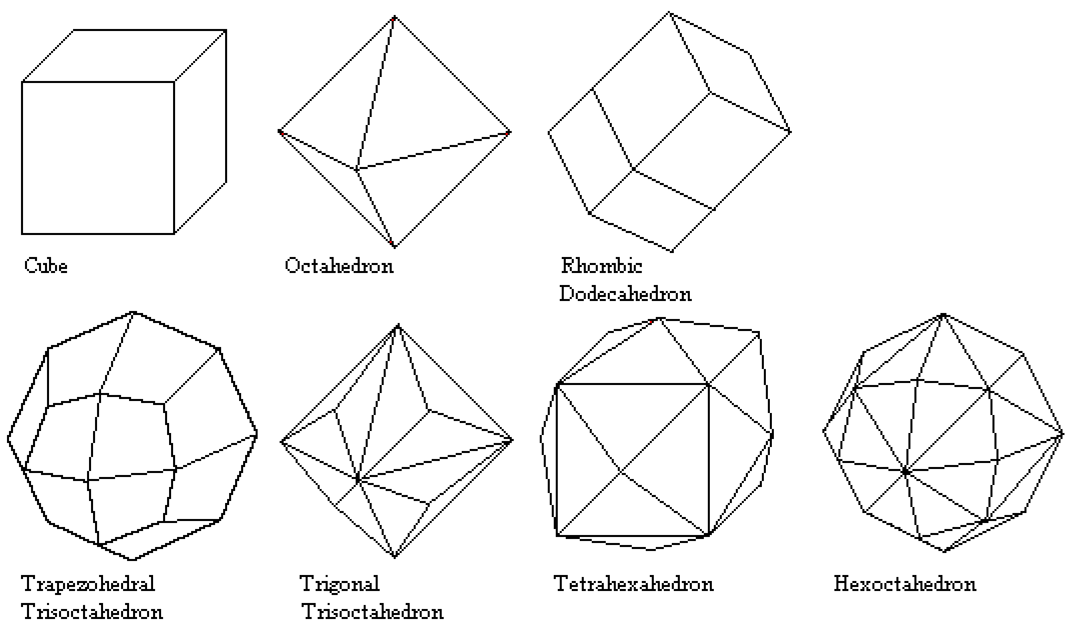
24 isoceles triangle faces (hll). Like an octahedron with a low triangular pyramid built on each face.

Tetrahexahedron:

24 isoceles triangle faces (h0l). Like an cube with a low pyramid built on each face.

Hexoctahedron:

48 triangular faces (hkl) The general form



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Prisms:

1st order – Faces cut the a1,a2 crystallographic axes with same unit length.
2nd order – Faces cut only one horizontal crystallographic axes (a1 or a2)
3rd order - Faces cut the a1,a2 crystallographic axes with different unit lengths.

Rhombic Prism:

1st order – Faces cut the b,c axes and parallel to a axis
2nd order – Faces cut the a,c axes and parallel to b axis
3rd order – Faces cut the a,b axes and parallel to c axis
4th order – Faces cut the all three a,b,c axes

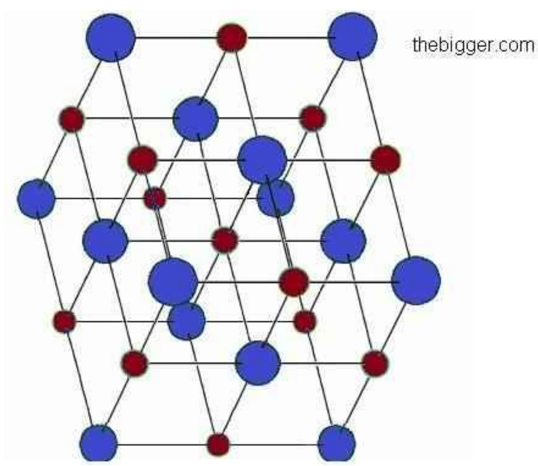
Pinacoids:

1st order – Faces cut the b,c axes and parallel to a axis
2nd order – Faces cut the a,c axes and parallel to b axis
3rd order – Faces cut the a,b axes and parallel to c axis
4th order – Faces cut the all three a,b,c axes

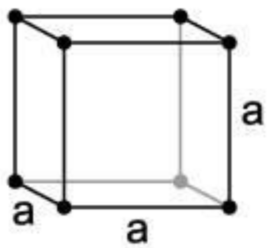
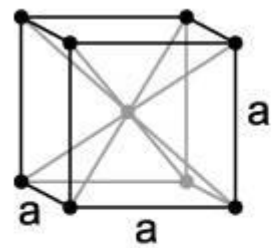
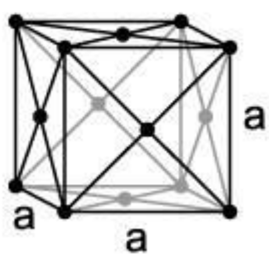
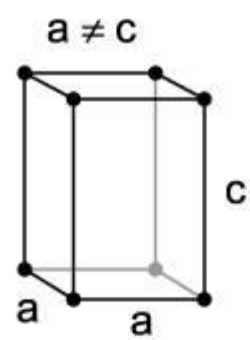
Crystal lattice

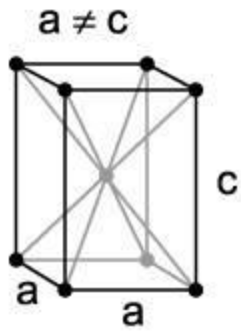
Crystals, and therefore minerals, have an ordered internal arrangement of atoms. This ordered arrangement shows symmetry, i.e. the atoms are arranged in a symmetrical fashion on a three dimensional network referred to as a ***lattice***.

To get a complete shape of solid, its atoms, molecules or ions must be placed at some particular places or points. A crystal lattice basically tells us about the basic structure of those points. After the correct placement of atoms on those points the original crystal structure is obtained. Each single point in a crystal lattice is known as lattice site or lattice point.

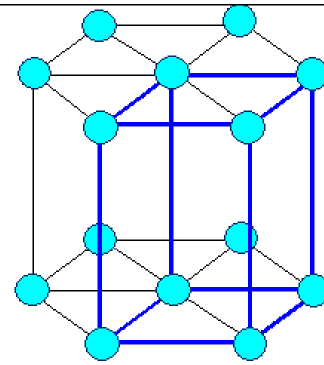


Space-Lattices: are produced by translations along three vectors. Here ions are repeated again with constant distances and angles in 3-D that produce 14 unique space-lattices, **which are also known as Bravais Lattices.**

<u>Simple Cubic or Isometric Bravais Crystal Lattice</u> $a_1 = a_2 = a_3$; $\alpha=\beta=\gamma=90^0$  The simple cubic system has one lattice point on each corner of the cube with each lattice point shared equally between eight adjacent cubes.	<u>Body-Centered Cubic Bravais Crystal Lattice</u> $a_1 = a_2 = a_3$; $\alpha=\beta=\gamma=90^0$  The body centered cubic system has eight corner points and one lattice point in the center of the unit cell.
<u>Face-Centered Cubic Bravais Crystal Lattice</u> $a_1 = a_2 = a_3$; $\alpha = \beta = \gamma = 90^0$  The lattice points of a face-centered cubic crystal are on the faces of the cube.	<u>Simple Tetragonal Bravais Crystal Lattice</u> $a_1 = a_2 \neq c$ $\alpha = \beta = \gamma = 90^0$  Tetragonal crystal lattices result from stretching a cubic lattice along one lattice vectors, making the cube a rectangular prism with a square base.
<u>Body-Centered Tetragonal Bravais Crystal Lattice</u> $a_1 = a_2 \neq c$ $\alpha = \beta = \gamma = 90^0$	<u>Hexagonal Bravais Crystal Lattice</u> $\mathbf{a_1 = a_2 = a_3 \neq c}$ $\boldsymbol{\alpha = \beta = \gamma \neq 90^0}$ **$\alpha = \beta = 90°, \gamma = 120°$**



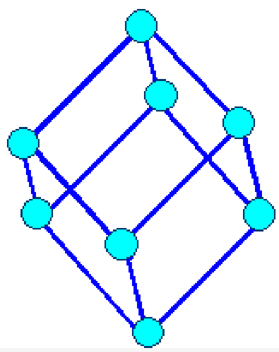
Tetragonal crystal lattices are made by stretching a cubic lattice along one of its lattice vectors, making the cube a rectangular prism with a square base.



A hexagonal lattice has the same symmetry as a right prism with a hexagonal base. Graphite is an example of a hexagonal crystal.

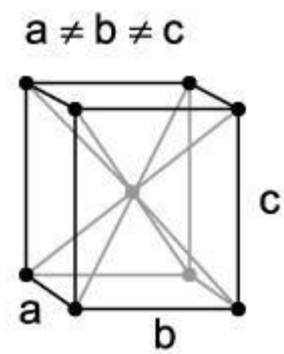
<u>Rhombohedral or Trigonal Bravais Crystal Lattice</u>

$a_1 = a_2 = a_3 \neq c \quad \alpha = \beta = \gamma \neq 90^0$



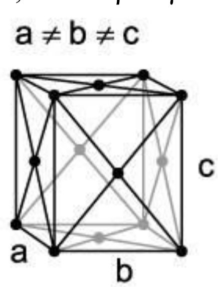
The rhombohedral (or trigonal) crystal is described by vectors of equal length, all three of which are not mutually orthogonal. You can think of the rhombohedral system as the cubic system stretched along a body diagonal.

<u>Body-Centered Orthorhombic Bravais Crystal</u>
Lattice $a \neq b \neq c; \quad \alpha = \beta = \gamma = 90^0$



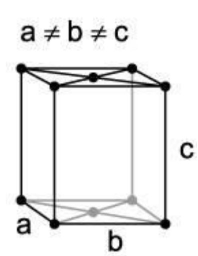
Orthorhombic lattices are made by stretching a cubic lattice along two lattice vectors by two factors, forming a rectangular prism with a rectangular base. All three bases intersect at 90° angles and the three lattice vectors are mutually orthogonal.

<u>Face-Centered Orthorhombic Bravais Crystal</u>
Lattice $a \neq b \neq c; \quad \alpha = \beta = \gamma = 90^0$



Orthorhombic lattices are made by stretching a cubic lattice along two lattice vectors by two factors, forming a rectangular prism with a rectangular base. All three bases intersect at 90°

<u>Base-Centered Orthorhombic Bravais Crystal</u>
Lattice $a \neq b \neq c; \quad \alpha = \beta = \gamma = 90^0$



Orthorhombic lattices are made by stretching a cubic lattice along two lattice vectors by two factors, forming a rectangular prism with a rectangular base. All three bases intersect at 90°

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<td>angles and the three lattice vectors are mutually orthogonal.</td>
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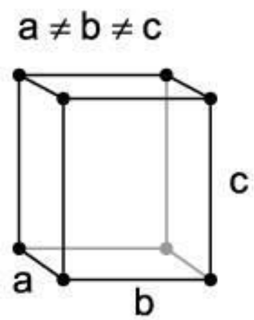
<u>Simple Orthorhombic Bravais Crystal Lattice</u>
a ≠ b ≠ c; α = β = γ = 90⁰



Orthorhombic lattices are made by stretching a cubic lattice along two lattice vectors by two factors, forming a rectangular prism with a rectangular base. All three bases intersect at 90° angles and the three lattice vectors are mutually orthogonal.
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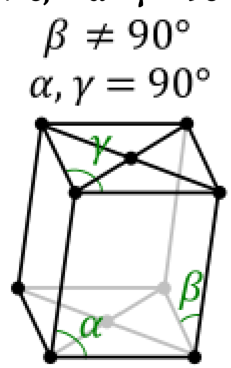
<u>Base Centered Monoclinic Bravais Crystal Lattice</u> **a ≠ b ≠ c; α= γ = 90⁰ ; β ≠ 90⁰**



A monoclinic lattice is described by vectors of unequal length that form a rectangular prism with a parallelogram as base. Two pairs of vectors are perpendicular, while the third pair makes an angle other than 90°.
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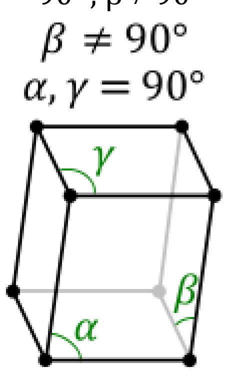
<u>Simple Monoclinic Bravais Crystal Lattice</u>
a ≠ b ≠ c; α= γ = 90⁰ ; β ≠ 90⁰



A monoclinic lattice is described by vectors of unequal length that form a rectangular prism with a parallelogram as base. Two pairs of vectors are perpendicular, while the third pair makes an angle other than 90°.
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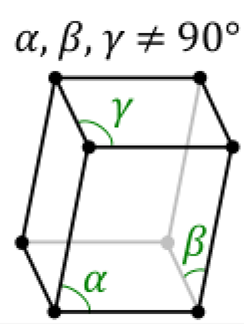
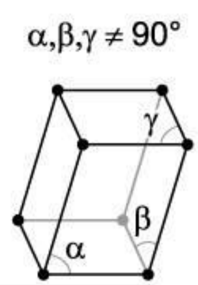
<u>Triclinic Bravais Crystal Lattice</u>
a ≠ b ≠ c; α ≠ β ≠ γ ≠ 90⁰





A triclinic crystal is described by vectors of unequal length. All three vectors are not mutually orthogonal.
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CLASSIFICATION OF CRYTALS

There are thousands of crystals, each showing characteristic morphology. The significance of crystal morphology can only be understood if at least one common property to all crystals can be used as the basis of classification. This common property is symmetry. The characteristic feature of symmetry is Repetition. A crystal may show repetition with respect to a point (centre of symmetry), a line (axis of symmetry) or/ and a plane (plane of symmetry). Every crystal is characterized by a specific combination of symmetry elements or by total absence of symmetry.

In 1830 Hessel, a German crystallographer showed by mathematical reasoning that there could be altogether 32 classes of crystals. Each class is characterized by a specific combination of symmetry elements. Of these 32 classes, some are represented by artificially prepared compounds, some by rare minerals and some without any representatives. The common minerals are found in eleven groups only. Therefore the first classification of crystals is into symmetry classes.

All types of unit cells (unit cell is the smallest complete unit of pattern giving the idea of crystal structure) belong to six sets of axes that are used as crystallographic axes. All the crystal forms, of whatever symmetry, that can refer to the same set of crystallographic axes belong to one crystal system. Therefore the next classification of crystals is onto crystal systems.

All crystal systems can be placed in one of the above seven systems, simply by determining their external symmetry elements. However, not all crystals in any system have the full or normal symmetry of that system. Some mineral species form crystals with less symmetry than the normal for their system. When this factor is considered, additional classes must be made under each major system to accommodate crystals of lower than normal symmetry, giving with the holosymmetric classes, a total of 32 sets(classes) of symmetry combinations.

S.No.	System	Number of Classes
1	Isometric (cubic)	5
2	Tetragonal	7
3	Hexagonal	7

4	Trigonal	5
5	Orthorhombic	3
6	Monoclinic	3
7	Triclinic	2

In any system the class with the highest symmetry is represented by the largest number of minerals. The percentages of minerals crystallizing in different systems are: triclinic-7.4; **monoclinic-31.7**; **orthorhombic-22.3**; trigonal-9.0; hexagonal-7.6; tetragonal-9.8; isometric (cubic) -12.2. This shows that more than 50% of all known minerals crystallize in two systems, monoclinic and orthorhombic.

Lattice: Crystals, and therefore minerals, have an ordered internal arrangement of atoms. This ordered arrangement shows symmetry, i.e. the atoms are arranged in a symmetrical fashion on a three dimensional network referred to as a ***lattice***.

Space-Lattices: are produced by translations along three vectors. Here ions are repeated again with constant distances and angles in 3-D that produce 14 unique space-lattices (point system), **which are also known as Bravais Lattices.** Basis on lattice symmetry the crystals are classified into **7 Systems.**

Unit cell: The smallest (fundamental) part of a crystal is called as unit cell. It is formed by combination of regularly ordered arrangements of atoms and molecules (lattice). The whole crystal structure can be formed by the repetition of these unit cells. In other words, the unit cell defines the basic building blocks of the crystal.

Pointgroup: In crystallography, a crystallographic point group is **a set of symmetry operations**, like rotations or reflections, that leave a central point fixed while moving other directions and faces of the crystal to the positions of features of the same kind. Basis on symmetry elements (Point groups) the crystals are classified into **32 classes.**

In studying crystals it is necessary to refer the forms to the crystallographic axes, and it has already been noted that there are various types of these axes. All the crystal forms that can be referred to the same set of crystallographic axes fall in one crystal “system”. This classification of crystals is therefore into “Crystal Systems” (Six Systems). So, Basis on the axes of reference (reference axes) i.e. crystal elements or axial elements (axial ratio & axial angle) the crystals are classified into **6 Systems.**

Symmetry of Normal Classes of 7 Systems

System	Plane of Symmetry	Axes of Symmetry	Centre of Symmetry	No. of Symmetry Elements
Cubic System [Isometric System]: a = b = c; $\alpha = \beta = \gamma = 90^0$	13 (3^{iv} , 4^{iii} & 6^{ii})	9 (3 axial & 6diagonal)	Present	23
Tetragonal system $a = b \neq c$; $\alpha = \beta = \gamma = 90^0$	5 {3 axial (1H;2V) & 2 (V) diagonal}	5 {4^{ii} (H) & 1^{iv} (Vert.Cryst. Axis}	Present	11
Hexagonal system $a_1 = a_2 = a_3 \neq c$; (it has 4-axes) $\beta_1 = \beta_2 = \beta_3 = 90^0$, $\gamma_1 = \gamma_2 = \gamma_3 = 120^0$ In this the Vertical Crystallographic axis is 6 – fold.	7 (4 axial & 3 diagonal)	7 {6^{ii} (H) & 1^{vi} (Vert.Cryst. Axis)}	Present	15
Trigonal system $a_1 = a_2 = a_3 = c$; (it has 4-axes) $\beta_1 = \beta_2 = \beta_3 = 90^0$, $\gamma_1 = \gamma_2 = \gamma_3 = 120^0$ In this the Vertical Crystallographic axis is 3 – fold.	3 vertical diagonal	4 {3^{ii} (H) & 1^{iii} (Vert.Cryst.Axis)}	Present	8
Orthorhombic system $a \neq b \neq c$; $\alpha = \beta = \gamma = 90^0$	3 Axial	3^{ii} Cryst. Axes	Present	7
Monoclinic system $a \neq b \neq c$; $\alpha = \gamma = 90^0$ $\beta \neq 90^0$	1 (contain **a** & **c** axis)	1^{ii} (Ortho axis)	Present	3
Triclinic system $a \neq b \neq c$; $\alpha \neq \beta \neq \gamma \neq 90^0$	None	None	Present	1

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Parameters

The ‘Parameters’ of a crystal face are the ratios of the distances from the origin at which the face cuts the crystallographic axes - that is the parameters are the ratios of the intercepts.

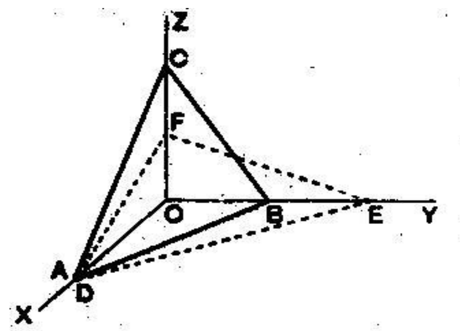


FIG. 31. Parameters.

In the fig. OX, OY, OZ represent the crystallographic axes, ABC is a crystal face making intercepts of OA on OX, OB on OY, OC on OZ. The parameters of the face ABC are given by the ratio of OA, OB and OC. It is convenient to take the relative intercepts of this face as standard lengths for the purpose of representing the position of any other face, such as DEF. In the case of the face DEF. OD is equal to OA, OE is twice OB, and OF in half OC, and therefore 1/1, 2/1, 1/2 are the parameters of DEF, with reference to the standard face ABC.

The form whose face is taken as intersecting the axes at the unit lengths which are to be used for measuring the intercepts made by other forms on the same axes is called the “fundamental, parametral or unit form”.

The parameters of the unit form can be obtained by measurement and can be expressed as multiples of one of their numbers.

Indices: - The reciprocals of the parameters are called the indices and are of use for purposes of crystallographic notation.

Lettering and order of the crystallographic Axes:- There are certain conventions with regard to the lettering and order of the crystallographic axes. The crystallographic axis which is taken as the vertical axis is called “c”, that running from right to left is “b”, and that running from front to back is “a”. One end of each axis is positive, and the other end is negative. **The angle between + a and + b is called γ, that between + b and +c is called α, and that between + c and +a is β.**

The planes in which two of the crystallographic axes lie are called the “Axial planes”.

Crystallographic Notation: Crystallographic notation is a concise method of writing down the relation of any crystal face to the crystallographic axes. The most widely used systems depend upon either

parameters or indices. Of these systems of notation, the chief are two - the **“Parameter system of Weiss”** and the “**Index system of Miller**” (modified by Bravais).

Parameter system of Weiss: - In this system of crystallographic notation, the axes are taken as a, b, c for unequal axes, a, a, c for two axes equal; a, a, a for three axes equal. The intercept that the crystal face under discussion makes on the a-axis is then written before a, the intercept on the b-axis before b, and the intercept on the c-axis before c. These intercepts are of course measured in terms of the intercepts made by the unit form on the corresponding crystallographic axes.

The most general expression for a crystal face in the weiss notation is-

na, mb, pc

where *n, m* and *p* are the lengths cut off by the face on the a, b, c axes as compared with the corresponding lengths cut off by the unit form. It is usual to reduce either n or m to unity. If a crystal face is parallel to an axis, it can be imagined as cutting that axis at an infinite distance, and accordingly the sign of infinity ∞, is placed as its parameter before the corresponding axial letter. Eg, 1 a, 2 b, ∞c (1a : 2b: ∞c)

Index System of Miller: - In this system of notation the indices or reciprocals of the parameters, are used. They are written in the axial order, a, b, c, and are always given in their most simple form by clearing of fractions. Eg Weiss symbol is 1 a, 2 b, ∞c. The reciprocals of the parameters are:-

1/1 a, 1/2b, 0c(1/∞c)

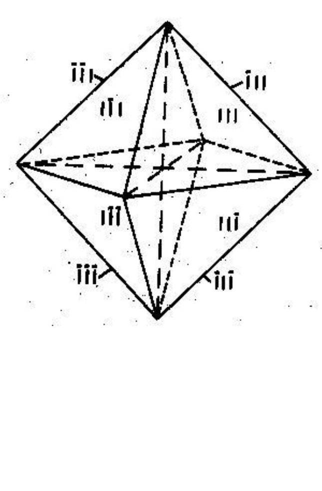
Clearing of fractions and omitting the axial letters the miller symbol is obtained -

210 which is read as two one nougat.

Conventions in Notation:- when it is required to indicate a crystal “form”, it is usual to enclose the symbols in a bracket thus (h k l), whereas, if the crystal “face” is indicated, the bracket is removed, thus hkl. By some, however, the form is enclosed in a curly brackets or brace {hkl}, and the face in an ordinary bracket (hkl).

A face cutting the positive end of an axis is indicated by the corresponding index figure only, whilst one cutting the negative end has a negative sign placed above the index figure.

Eg. Octahedron 111, $11\bar{1}$, $1\bar{1}1$, $1\bar{1}\bar{1}$, $\bar{1}11$, $\bar{1}1\bar{1}$, $\bar{1}\bar{1}1$, and $\bar{1}\bar{1}\bar{1}$.



The law of Rational Indices: - (Intercepts): The study of crystals has established the law of rational indices which states that "the intercepts that any crystal face makes on the crystallographic axes are either infinite or small rational multiples of the intercepts made by the unit form".

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Brief notes on the Normal class of isometric system

RANJITA

Axial relationship:

Crystals belonging to isometric or cubic system are referred to three crystallographic axes, which are equal in length and are perpendicular to each other.

Since these are mutually interchangeable, they are all designated by letter 'a'. However, to distinguish them from each other and for proper orientation, they are designated as a, a_2 and a. In proper orientation an axis runs from front to back, positive on observer side and negative on the other side; a_2 axis runs from right to left, positive on the right hand side and negative on left hand side and a_3 axis is vertical, positive at the top and negative at the bottom.
Axes a, and a, lie in the horizontal plane and a_3 is vertical and perpendicular to the plane containing the, and a_2 axes. Since all the crystallographic axes are equal in length the axial ratio (a: b: c) in case of isometric system is equal to **five classes** are grouped under this system.

These are hex octahedral, hex tetrahedral, diploidic, gyroidal and tetartoidal. The hex octahedral class is the normal class of isometric system as it shows maximum symmetry among all classes and also as galena type after the characteristic mineral galena that crystallizes in this class.

Symmetry elements:

Crystals belonging to this class are characterized by three axes of 4-fold symmetry, which are coincident with the crystallographic axes. There are four axes of 3-fold symmetry, which emerge in the middle of each of the octants formed by intersection of crystallographic axes.

In addition, there are six axes of 2-fold rotation, each of which bisects one of the angles between two crystallographic axes. There are nine mirror planes, three axial i.e. each contains two of the crystallographic axes and six diagonal each of which bisects the angle between a pair of the axial planes. Centre of symmetry is also present.

Forms:

<u>The Closed Forms:</u>

The Isometric Forms

The Cube

The cube is familiar to everyone as a symmetrical six sided box. Although the cube is familiar it is hard to identify its faces on a highly modified crystal that has faces from other forms. Remember it has eight points, six faces and twelve edges that are perpendicular to each other forming 90 degree angles and square cross-sections. The faces are usually square, but if modified can be other geometric shapes. The cube can only be formed by isometric minerals. **Galena** is a mineral that forms cubes.

The Octahedron

The octahedron is a symmetrical eight sided shape that may look like two four sided pyramids lying base to base. But closer inspection will show that each set of opposing points is exactly the same and could serve equally well as the "top" and "bottom" of the two pyramids. In fact there is no top or bottom on isometric forms. The faces are equilateral triangles unless modified. There are six points, eight faces and twelve edges. Each face is parallel to the opposite face. The octahedron is related to the cube by placing each point of an octahedron at the center of each face of a cube. The mineral **diamond** commonly forms octahedrons.

The Tetrahedron

The tetrahedron has only four equilateral triangular faces (unless modified), four points and six edges and when sitting on one face looks like a trigonal pyramid. But every point of the tetrahedron can serve as the top of the pyramid because all four points are identical. The tetrahedron is related to the **octahedron** by placing the six points of the octahedron in the middle of the six edges of the tetrahedron. In fact the four faces of the tetrahedron would be parallel to four of the eight faces of the octahedron. The tetrahedron can be thought of as having removed every other face from an octahedron and extending the four faces that are left to complete the closed tetrahedron. Tetrahedrons only form in isometric minerals that have four fold rotoinversion axes. The mineral **tetrahedrite** as its name infers, forms mostly tetrahedrons.

Dodecahedrons

Another isometric shape the dodecahedron is twelve sided and there are four basic types. The first dodecahedron has symmetrical pentagonal faces (five edged polygons). The second dodecahedron has delta shaped faces (four edged polygons). The third dodecahedron has asymmetrical pentagonal faces. The last dodecahedron has rhombic faces (four edged diamond-shaped polygons).

The pentagonal dodecahedron is termed a pyritohedron because it is most commonly found in the mineral **pyrite**. The pyritohedron is related to the **cube** by orienting two of the pyritohedron's faces essentially with each cube face. Two of the pentagonal faces touch bases in a line or edge that is parallel to the cube faces. This edge is responsible for the striations seen on pyrite's cubic faces when the two forms are trying to form together.

The deltoid dodecahedron has four sided delta shaped faces. It is derived from a tetrahedron with each of its four faces replaced by three delta shaped faces. The deltoid dodecahedron is a rare dodecahedron and is usually only seen modifying the tetrahedral crystals of **tetrahedrite** and **tennantite**.

The tetartoid is a 12 sided form that is very rarely seen. The faces are asymmetrical pentagons. The mineral **cobaltite** has been known to form this type.

The rhombic dodecahedron is found commonly in the **garnets** as well as other minerals including **fluorite** (pictured above left on **rhodochrosite**). Each of the faces of this dodecahedron have a parallel face across from them. The faces are rhombic or diamond-shaped, like the diamonds in a deck of cards. A rhombic dodecahedron has fourteen points, twelve faces, and twenty four edges. A rhombic dodecahedron is related to the **octahedron** by placing a dodecahedral face over each of the twelve octahedral edges. The generic use of the word dodecahedron is usually applied to this rhombic form.

The Gyroid

The gyroid has 24 non-symmetrical pentagonal faces and is a rare form. **Cuprite** is the only mineral that commonly forms gyroids and usually they are modified by other forms.

The Diploid

The diploid is also a form that is rarely seen. It has 24 four edged faces that are formed from the splitting of the pentagonal faces on the 12 sided **pyritohedron**. The split is from the center of the pentagons base edge to the top point of the pentagon. The mineral **skutterudite** will occasionally form this rare habit.

The Hexoctahedron

The hexoctahedron is a richly faceted form with a total, if fully formed, of 48 triangular faces. It is related to the octahedron by the dividing of each of the octahedron's eight faces into six triangular faces. **Diamond** is found in this form however the crystals tend to appear rounded.

The Tetrahexahedron

This form is composed of 24 triangular faces and is related to the **cube** by dividing each of the cubes six faces into four faces. The edges of the cube remain in place and form the bottom edge of each isosceles triangle. This form will still give an overall cubic look but with each cube face pushed outward to a four sided squat pyramid. The mineral **fluorite** has shown this form but the tetrahexahedral faces are usually just modifying the cubic faces.

The Trapezohedron

The trapezohedron has 24 trapezium or deltoid shaped faces. They can be thought of in two ways. Either they are dividing each face of an **octahedron** into three faces or they are dividing each face of a **cube** into four faces. Either way the result looks a little more like a cube than an octahedron however no edges of the cube or the octahedron remain. The mineral **analcime** is commonly seen in this form.

The Tristetrahedron

The tristetrahedron has 12 faces that are shaped like extremely acute isosceles triangles. It is formed by the dividing of a **tetrahedral** face into three faces. The original edges of the tetrahedron serve as the bases for the triangular faces of the tritetrahedon. The mineral **sphalerite** can have faces of the tristetrahedron.

The Trisoctahedron

The trisoctahedron has 24 faces that are shaped like extremely acute isosceles triangles. It is formed by the dividing of an **octahedal** face into three faces. This is different from the effect created in the **trapezohedron** in that these faces leave the original edges of the octahedron in place and use them as their base for their squashed triangular shapes. The mineral **Diamond** can have octahedral crystals modified by the trisoctahedron but rarely is this form fully developed.

The Hextetrahedron

This is also a rare form that almost is never fully developed. It is composed of 24 triangular faces and is formed from the dividing of each face of the **tetrahedron**into six faces. Its faces are sometimes seen on the highly modified crystals of the mineral **sphalerite**.