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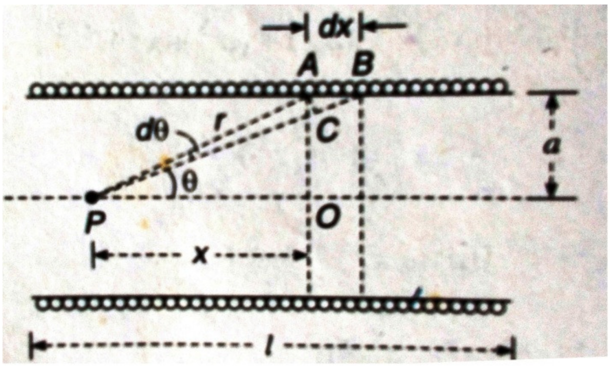
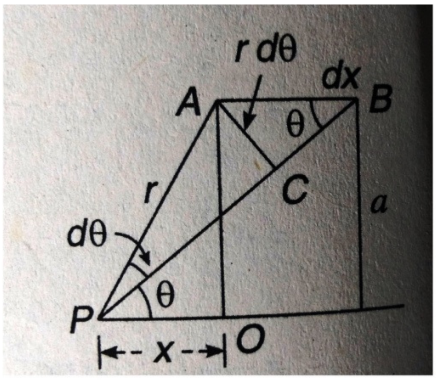
MAGNETIC FIELD INDUCTION DUE TO A SOLENOID

(సోలనాయిడ్ వల్ల అయస్కాంత ప్రేరణ)

Consider a long solenoid of length 'l' metre , radius 'a' metre as shown in the following figure. Let 'N' be total number of turns in the solenoid in which a current 'i' passes through it . The number of turns per metre 'n' will be N/l.





The aim of this topic is to calculate the magnetic field induction due to this solenoid in the following cases.

1. Magnetic field at an inside point (లోపలి బిందువు వద్ద అయస్కాంత ప్రేరణ)

2. Magnetic field at an axial end point (అక్షీయ అంత బిందువు వద్ద అయస్కాంత ప్రేరణ)

3. Magnetic field at the centre of solenoid of finite length (సోలనాయిడ్ లోపలి మధ్య బిందువు వద్ద అయస్కాంత ప్రేరణ)

Case I: Magnetic field at an inside point

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In this case, we shall calculate magnetic induction at a point 'P' on the axis as shown in the above. For this purpose, we divide the solenoid into a number of small equidistant coils. Consider one such coil of width 'dx'with ndx turns. The magnetic induction 'B' at point 'P'due to the coil of width 'dx' carrying a current 'i' is given by

$$dB == \frac{\mu_0 (ndx) ia^2}{2[a^2+x^2]^{\frac{3}{2}}} \frac{weber}{metre^2} \qquad \text{------(1)}$$

From the above figure, using the triangle ABC, one can write,

$$sin\theta = \frac{rd\theta}{dx} or\ dx = \frac{rd\theta}{sin\theta}$$

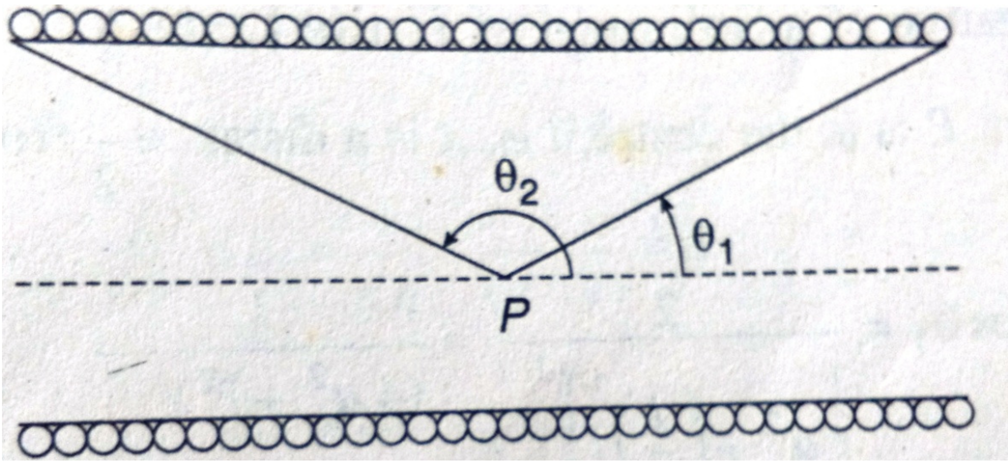
Further, from triangle APO, $a^2 + x^2 = r^2, (a^2 + x^2)^{\frac{3}{2}} = r^3$

Substituting these values in equation (1), we get,

$$dB == \frac{\mu_0 n(r\frac{d\theta}{sin\theta})ia^2}{2r^3} \Longrightarrow dB = \frac{\mu_0 nia^2 d\theta}{2r^2 sin\theta} = \frac{\mu_0 nid\theta}{2sin\theta}\left(\frac{a}{r}\right)^2 = \frac{\mu_0 nid\theta}{2sin\theta}(sin\theta)^2$$

$$\therefore dB = \frac{\mu_0 nid\theta sin\theta}{2} \qquad \text{------(2)}$$

The magnetic induction 'B' at 'P' due to the whole solenoid can be calculated by integrating equation (2) between the limits θ_1 and θ_2. These angles are shown in the following figure.



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$\therefore B = \int_{\theta_1}^{\theta_2} \frac{\mu_0 n i d\theta sin\theta}{2} = \frac{\mu_0 ni}{2}[-cos\theta]_{\theta_1}^{\theta_2}$, $\therefore \boldsymbol{B = \frac{\mu_0 ni}{2}[cos\theta_1 - cos\theta_2]}$ **---------**(3)

At any axial point 'P' well inside a very long solenoid, $\theta_1=0$ and $\theta_2=\pi$

$$\therefore B = \frac{\mu_0 ni}{2}[cos0 - cos\pi] \Longrightarrow \boldsymbol{B = \frac{\mu_0 ni}{2}[1-(-1)] = \mu_0 ni}$$

Case II: Magnetic field at an axial end point

In this case, $\theta_1=0$ and $\theta_2=90^\circ$ or $B = \frac{\mu_0 ni}{2}[cos0 - cos90^\circ] = \frac{\mu_0 ni}{2}$

$$\therefore B = \boldsymbol{\frac{\mu_0 ni}{2}}$$

Case III: Magnetic field at the centre of solenoid of finite length

In this case, length of the solenoid is taken as 'l' and half the length is l/2.

$$\therefore cos\theta_1 = \frac{\frac{l}{2}}{\left[a^2 + \left(\frac{l}{2}\right)\right]^2} = \frac{l}{(4a^2+l^2)^{\frac{1}{2}}}$$

$$cos(\pi - \theta_2) = \frac{\frac{l}{2}}{\left[a^2 + \left(\frac{l}{2}\right)\right]^2} = \frac{l}{(4a^2+l^2)^{\frac{1}{2}}} \Longrightarrow \boldsymbol{cos\theta_2 = -\frac{l}{(4a^2+l^2)^{\frac{1}{2}}}}$$

Substituting these values in equation (3), one can get

$$B = \frac{\mu_0 ni}{2}\left[\frac{l}{(4a^2+l^2)^{\frac{1}{2}}} + \frac{l}{(4a^2+l^2)^{\frac{1}{2}}}\right] = \frac{\mu_0 ni}{2}\left[\frac{2l}{(4a^2+l^2)^{\frac{1}{2}}}\right] = \frac{\mu_0 nil}{(4a^2+l^2)^{\frac{1}{2}}}$$

$$\Longrightarrow \boldsymbol{B = \frac{\mu_0 iN}{(4a^2+l^2)^{\frac{1}{2}}}} \quad \textbf{-----}(4)$$

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where, nxl=N

Thus, equation (4) gives an expression for the magnetic field induction ‘B’ at the centre of the coil.

Model questions(మాదిరి ప్రశ్నలు):

1. Calculate the magnetic field induction due to a solenoid.
2. Using Biot - Savart law, derive an expression for the magnetic induction ‘B’ due to a solenoid.
3. State and explain Biot - Savart law. Derive an expression for the magnetic induction inside a solenoid carrying current i.
4. Show that the magnetic field at the ends of a solenoid is half of that in the middle.
5. Using Biot - Savart law, show that the magnetic field at the ends of a solenoid is half of that in the middle.

References:

1. Unified Physics, Volume III, Jai Prakash Nath Publications, Meerut

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