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E – CONTENT / STUDY MATERIAL

I BSc MECHANICS, WAVES AND OSCILLATIONS

PREPARED by: -

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UNIT 1

I. MECHANICS OF PARTICLES

(1) Review of Newton's laws of motion

Newton proposed three laws of motion.

- **First law:** Everybody continuous in its state of rest or of uniform motion in a straight line unless it is compelled by same external force to change that state.
- **Second law:** The rate of change of linear momentum of a body is directly proportional to the applied force and the change takes place in the direction of applied force.
- **Third law:** To every action, there is always an equal and opposite reaction.

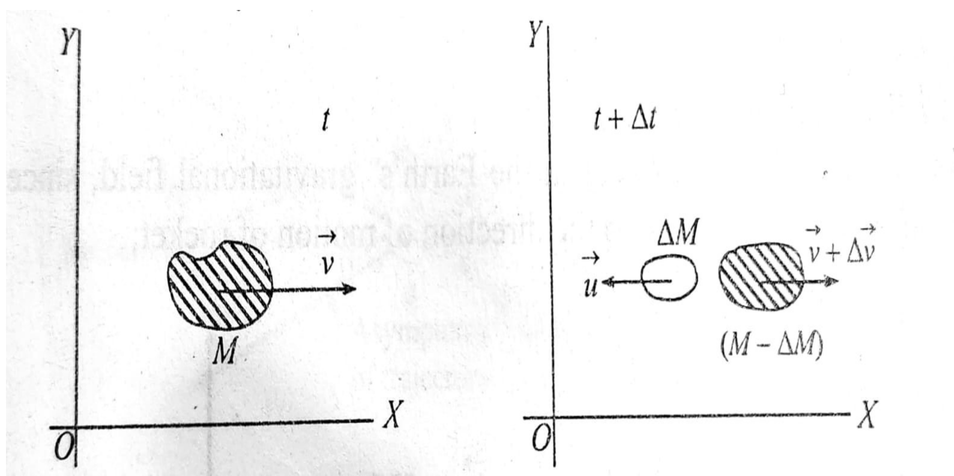
Limitations:

1) The second law (F = ma) is not applicable when the mass 'm' is variable. E.g. In Rocket motion.
2) When the particles move with relativity velocities, second law is not applicable.
3) The third law is not applicable for particles of atomic size. Because the action and reaction cannot be measured simultaneously when the particles are of atomic size.

(2) Motion of variable mass system

System of Variable Mass (Motion of a rocket)

1) Rocket is a mass varying system. At an instant let M be the mass of the rocket and $\vec{v}$, its velocity relative to the earth.



2) In a small interval of time Δt, let Δ M be the mass of the gases burnt which eject out of the rocket with a velocity, say $\vec{u}$ relative to the earth and $\Delta\ \vec{v}$ be the increase in the velocity of the rocket.

3) In this small time interval, total initial momentum of the system, $\overrightarrow{P_1} = M\vec{v}$

Total final momentum of the system, $\vec{P} = (m - \Delta M)$

4) Considering both parts of masses AM and M - AM are forming from the same system. We can write external force M on Rocket is, P2-P R

(3) Motion of a Rocket

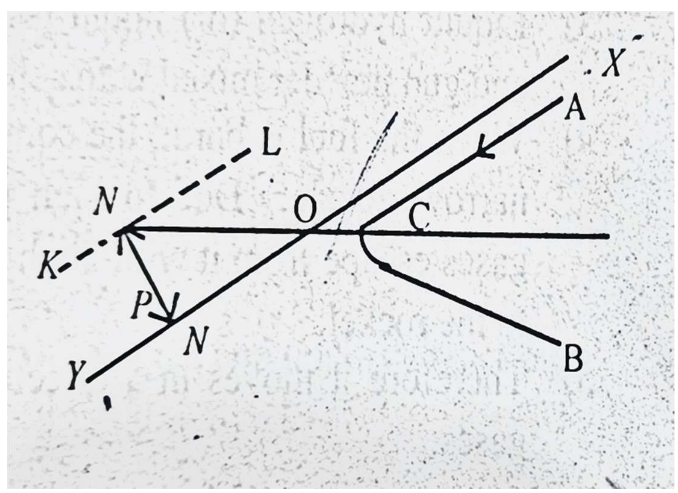
- The rocket motion is based on the principle of conservation of linear momentum
- It consists of a fuel chamber in which the fuel may be in the form of liquid (or) solid is stored.
- Liquid hydrogen (or) liquid paraffin with solid oxidiser forms the liquid fuel and gun powder mixed with solid oxidiser form the solid fuel.
- When the fuel is burnt, the combustion gases are allowed to escape through a narrow nozzle. Due to high pressure inside the combustion chamber, the gases escape from it with high exhaust velocities, importing large momentum to the rocket.
- Therefore, it moves in a direction opposite to the direction of escape of the gases.

(4) Multistage Rocket

- The escape velocity on earth v_e is 11.2 km/s and the orbital velocity v_0 is 8 km/s.
- To attain the speeds of this order a single stage rocket is not sufficient. To increase the speed of the rocket the expelled gases coming out in the form a a jet must have maximum velocity and the mass of the final stage rocket must be very much less than its mass the begining.
- The maximum attainable velocity of jet of gases in practice is around 2 km/s.
- To increase the velocity even further multistage rockets are used.
- In first stage when the fuel is completely burnt it detaches from rocket and drops off. The velocity acquired at the end of the first stage becomes the initial velocity of the second stage.
- The second stage is then ignited and starts functioning. The rocket gains acceleration and again when the fuel gets exhausted it also gets detached from rocket.
- Finally in the last stage the rocket gets the required velocity and only pay load remains which is used to conduct experiments.

(5) Concept of Impact Parameter & Scattering and Cross – Section

1) Impact parameter is defined as the closet distance between nucleus and positively charged particle projected towards it.



i) let xy be the initial direction of motion of a charged Alpha particle projected towards the targeted nucleus N.

ii) let KL be the line drawn parallel to xy and passing through N.

iii)The perpendicular distance P between the initial direction of the charged particle and the line parallel to this and passing through the target nucleus is called impact parameter P of the collision.

iv) if P=0 the collision becomes head on.

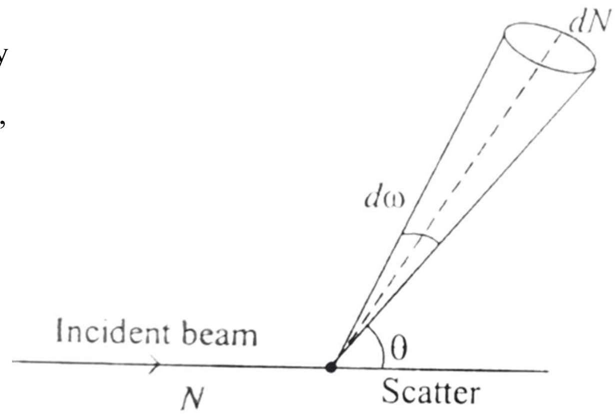
2) **Scattering cross section**

It is defined as the ratio of number of particles in the solid angle (dw) per unit time to the incident intensity

i) when a particles are projected into a thin metal foil, they are scattered in different directions

ii) Let N be the incident intensity. dN be the number of particles scattered per unit into solid angle dw

iii) the ratio $\frac{dN}{N}$ is called scattering cross section.



(6) Rutherford Scattering – Derivation

Rutherford scattering cross section:

1) Let an a-particle moving along AO, approach the stationary nucleus at N. Since both are positively charged, there is a force of repulsion between them and this force increases as they approach each other.

2) Applying the theory of central orbits, it can be shown, that a particle describes a hyperbola ACB, whose one focus is at N and whose asymptotes AO and BO give initial and final directions of motion of the a-particle.

3) Let m = mass of the a-particle,

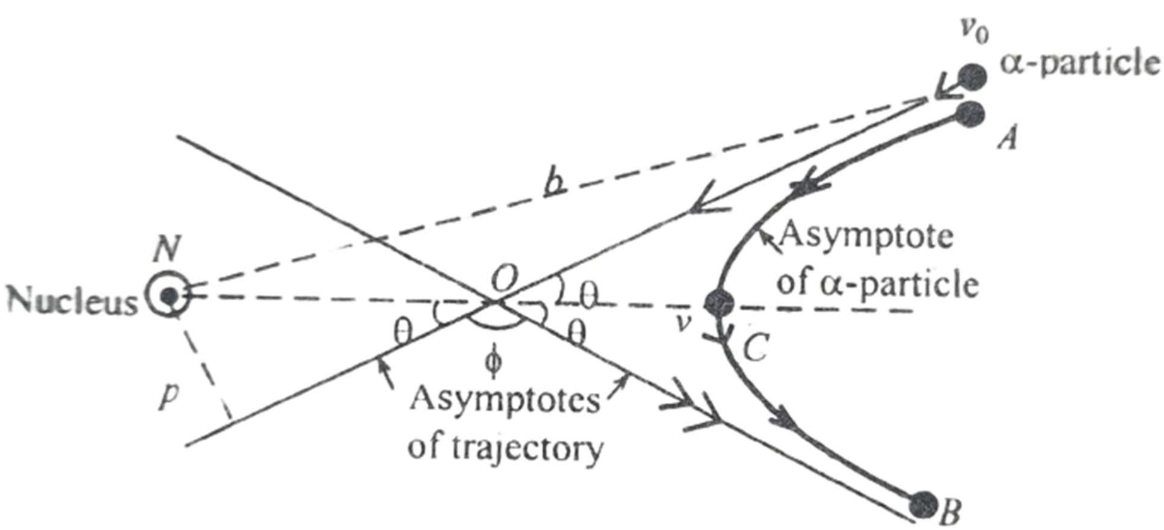
2e = charge on the a-particle

ze = charge on the nucleus, where z is the atomic number

ϕ= angle of scattering,

p = impact of parameter

Vo = initial velocity of the a - particle.



4) If a-particle is directed straight towards N, then p = 0.

5) On account of repulsive force, it will be stopped at a distance 'b' from the nucleus N and made to retrace its path. In such a case, = 180°.

6) Electrostatic potential at a distance 'b' from the nucleus = $\frac{Ze}{4\pi\epsilon_0 b}$

So, P.E of the a-particle at distance b = $\frac{Ze.2e}{4\pi\epsilon_0 b}$

7) Since the a-particle is momentarily stopped at a distance b from the nucleus, all its K.E must be completely converted into P.E.

$$\frac{2\ Ze2}{4\pi\ \epsilon b\ 2} = \frac{1}{2}mv2$$

8) But in general, the a-particle cannot be directed exactly towards the nucleus, we must consider the case when p# 0. In such a case a-particle will be deflected through an angle which is less than 180° and will travel along the hyperbolic path ACB.

9) Let v be the velocity of the a-particle at the vertex C of the hyperbola. Then applying the law of conservation of energy and law of conservation of angular momentum at A and C.

II. MECHANICS OF RIGID BODIES

(1) Rigid Body definition

A. Rigid body: The solid body in which the relative distances between the particles remain unchanged under the action of any external force is called a rigid body. It does not change its shape or size under the influence of any external force.

Equation of motion for a rotating rigid body:

1) Consider a rigid body is rotating a fixed axis passing through 'O' with a constant angular velocity ω. All the particles of the body have same angular velocity ω.

2) Every particle in the body moves in a circle with its centre on the axis of rotation.

3) Let us take a particle of mass 'm' at 'p'. Let its position vector be $\vec{r}$ with respect to 'O'.

4) The linear velocity of the particle $V = 0 \times 7$.

The momentum of the particle $p = mv = m(x7)$

5) The angular velocity of the whole body is given by L=Exp→ (1)

6) As shown in fig, let $SOP = 0$. The component of along the axis of rotation

will be $r \cos \theta$. Hence the magnitude of L is,

$$L = Em[ro - (ro\ cos\ e)\ (r\ cos\ 0)]$$

$$= Em[ro - ro\ cos^2 0]$$

:. $L = Emro \sin^2 0$

7) But $sine\ R = r\ sine = R$

$$=> L = 2mR2 = Io\ where\ I = EmR2$$

Where R is 'distance from the particle to the axis of rotation of the body.

8) The rate of change of angular momentum is called torque (t).

9) But $dL = dt\ dt\ do\ dt = a$. (Angular acceleration). t = Iα

The above equation is called the equation of motion of a rigid body rotating about the axis of symmatry (rotation).

(2) Rotational kinematic relations:

1) The equations of a particle related to rotatory motion with constant angular acceleration are called rotational kinematic equations.

2) They are a) $\boldsymbol{\omega = \omega_0 at}$, b) $\theta = \boldsymbol{\omega_0 t + \frac{1}{2} at^2}$, c) $\boldsymbol{\omega^2 = \omega_0 + 2a\theta}$

a) Proof $\boldsymbol{\omega = \omega_0 at}$: Consider a particle rotating about a fixed axis. In small interval of time dt, the initial angular velocity $\boldsymbol{\omega_0}$, changes to $\boldsymbol{\omega}$. Let a be its angular acceleration and θ be its angular displacement.

By definition of Angular acceleration, $\boldsymbol{a = \frac{d\omega}{dt}}$ $\boldsymbol{d\omega = a\,dt}$

Integrating the above equation with proper limits

$$\int_{\omega_0}^{\omega} d\omega = \int_{t=0}^{t} a dt$$

$$[\omega]_{\omega_0}^{\omega} = a[t]_0^t$$

$$\omega - \omega_0 = a(t-0)$$

$$\omega = \omega_0 + at$$

(3) Equation of motion for a rotating body

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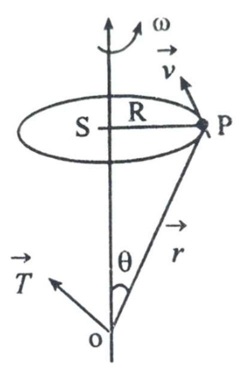
2) Every particle in the body moves in a circle with its centre on the axis of rotation

3) Let us take a particle of mass $'m'\ at\ 'p'$. Let its position vector be $\vec{r}$ with respect to 'O'.

4) the Linear velocity of the particle $\vec{V} = \vec{\boldsymbol{\omega}} \times \vec{r}$



The momentum of the particle $\vec{p} = m\vec{v} = m(\vec{\boldsymbol{\omega}} \times \vec{r})$

5) The angular velocity of the whole body is given by

$\vec{L} = \overrightarrow{\sum r} \times \vec{p} \rightarrow (1)$

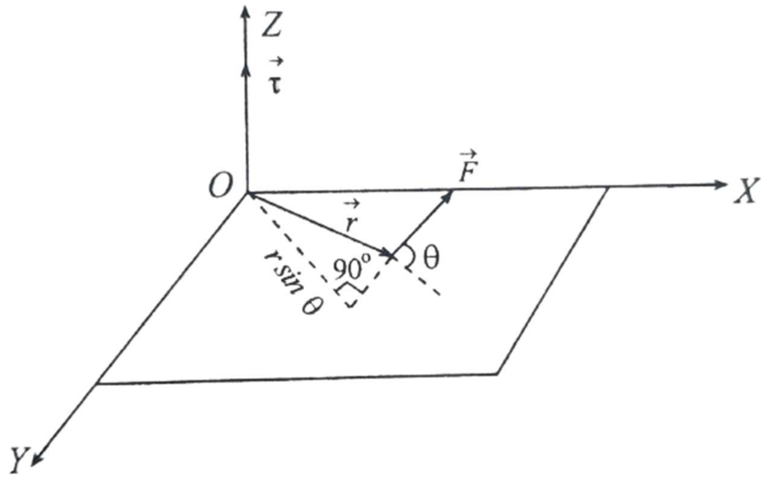
$\vec{L} = \overrightarrow{\sum r} \times\ m(\vec{\boldsymbol{\omega}} \times \vec{r})$ = $m\vec{r} \times \vec{\boldsymbol{\omega}} \times \vec{r}$

$$\vec{L} = \sum m[(\overrightarrow{r}.\vec{r})]\,\vec{\boldsymbol{\omega}} - (\vec{r}.\vec{\boldsymbol{\omega}})\,\vec{r}]$$

(4) Angular Momentum

It is defined as vector product of position vector $\overrightarrow{(r)}$ and momentum vector $\overrightarrow{(P)}$ i.e., $\vec{\boldsymbol{L}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{P}}$

Relation between angular momentum and torque:



1) Let a force $\vec{F}$ act on a particle P whose position vector $\vec{r}$ with respect to the origin in $'O'$ of an inertial reference frame.

2) The torque acting on a particle about origin O is defined as vector product of $\vec{r}$ and $\vec{F}$.

i.e., $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}} \rightarrow (\mathbf{1})$

3) From the definition of angular momentum, we can write $\vec{\boldsymbol{L}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{P}} \rightarrow (\mathbf{2})$

4) Differentiating equation (2), we get

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{P})$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{P}}{dt} + \frac{d\vec{r}}{dt} \times \vec{P}$$

But $\frac{d\vec{r}}{dt} \times \vec{P} = \frac{\overrightarrow{dr}}{dt} \times m\vec{v} = \frac{d\vec{r}}{dt} \times m\frac{d\vec{r}}{dt} = 0$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{P}}{dt}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} \rightarrow (3)\ where\ \vec{F} = \frac{d\vec{P}}{dt}$$

5) from equation (1) & (3), we get $\vec{\tau} = \frac{d\vec{L}}{dt}$. This is the relation between torque and angular momentum.

(5) Euler equations

Euler's equations:

1) The equation of motion of a rotating body is $t = dL/\,dt \rightarrow (1)$

2) One rotating frame can be transformed to another frame of reference using hungry operator i.e.,

$$\frac{\boldsymbol{d}}{\boldsymbol{dt}}(\ )\boldsymbol{space} = \frac{\boldsymbol{d}}{\boldsymbol{dt}}(\ )\boldsymbol{body} + \boldsymbol{\omega}\,(\ )$$

3) Let the hungry operator be applied on the angular momentum L,

$$\left(\frac{d\vec{L}}{dt}\right)_{\text{Space}} = \left(\frac{d\vec{L}}{dt}\right)_{\text{Body}} + \vec{\omega} \times \vec{L} \rightarrow (2)$$ where ω is angular velocity of rotating frame.

4) From equation (1) and (2), we get $\tau = \frac{d\vec{L}}{dt} + \vec{\omega} \times \vec{L} \rightarrow (1)$ Where $\vec{L}$ refers to the rotating frame.

5) The equations of τ, L and ω along the three axes are given by $\vec{\tau} = \tau_1 \hat{i} + \tau_2 \hat{j} + \tau_3 \hat{k}$

$\vec{L} = L_1 \hat{i} + L_2 \hat{j} + L_3 \hat{k}$ and $\vec{\omega} = \omega_1 \hat{i} + \omega_3 \hat{j} + \omega_3 \hat{k}$

6) But $\vec{\omega} \times \vec{L} = \begin{vmatrix} i & j & k \\ \omega_1 & \omega_2 & \omega_3 \\ L_1 & L_2 & L_3 \end{vmatrix}$

$$\Rightarrow \vec{\omega} \times \vec{L} = (\omega_2 L_3 - \omega_3 L_2)\,\hat{i} + (\omega_3 L_1 - \omega_1 L_3)\,\hat{j} + (\omega_1 L_2 - \omega_2 L_1)\,\hat{k}$$

7) The x- component of $\omega \times L = \omega_2 L_3 - \omega_3 L_2$
The y- component of $\omega \times L = \omega_3 L_1 - \omega_1 L_3$
The z- component of $\omega \times L = = \omega_1 L_2 - \omega_2 L_1$

8) From equation (2), the $X-$ component of torque, $\tau_1 = \frac{dL_1}{dt} + \omega_2 L_3 - \omega_3 L_2$

$$\tau_1 = \frac{dL_1}{dt} + \omega_2 I_3 \omega_3 - \omega_3 I_2 \omega_2 \quad (\because L = I\omega)$$

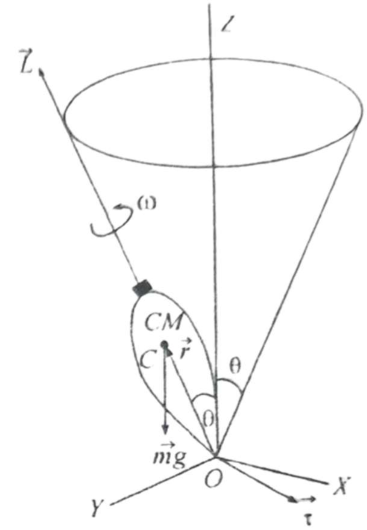
$$\tau_1 = \frac{dL_1}{dt} + (I_3 - I_2)\,\omega_2 \omega_3 \rightarrow (3)$$

6) Similarly, $\tau_2 = \frac{dL_2}{dt} + (I_1 - I_3)\,\omega_1 \omega_3 \rightarrow (4)$

$$\tau_3 = \frac{dL_3}{dt} + (I_2 - I_1)\,\omega_2 \omega_1 \rightarrow (5)$$

These (3), (4) and (5) equations are called Euler's equations.
When $\tau_1 = \tau_2 = \tau_3 = 0$ the body is said to be a torque free rotation.

(6) Precession of a spinning top



A. Precession of symmetric top:

1) A rigid body rotating about an axis of symmetry, which is fixed at one point, is called symmetric top.

2) As the top spins, it also rotates around Z-axis and the axis of symmetry sweeps a cone around Z-axis. This motion of the top is called 'precession'. Expression for precessional velocity of a symmetrical top:

3) Due to the precessional motion of the top let the angle made by the axis of symmetry with Z-axis be 0.

4) The directions of angular velocity o and angular momentum L of spin motion will be along axis of symmetry.

5) The weight of the top mg acts vertically downwards at the centre of mass 'C' of the top. This possesses a moment about the fixed point '0' and applies a torque t on the top.

6) If is the radius vector of the centre of mass, $t = r \times mg$. The scalar magnitude of $t = rmg\ sin\ (180° - 0) = rmg\ sine$ → (1)

7) The direction of will be perpendicular to both and mg obeying right hand screw rule. Its direction as shown in the figure will be perpendicular to Z.

8) Hence this torque does not change the magnitude of L but its direction changes into that T.

9) As the direction of Z changes, 7 also changes and at every instant will be perpendicular to Z.

10) As a result, the tip of the Z vector makes circular motion around Z-axis and L vector sweeps a cone (fig). This gives rise to the precession motion of the top. The relation between t & L is given by.

$$\boldsymbol{r} = \frac{\overrightarrow{\boldsymbol{dt}}}{\boldsymbol{dt}}$$

7) Gyroscope:

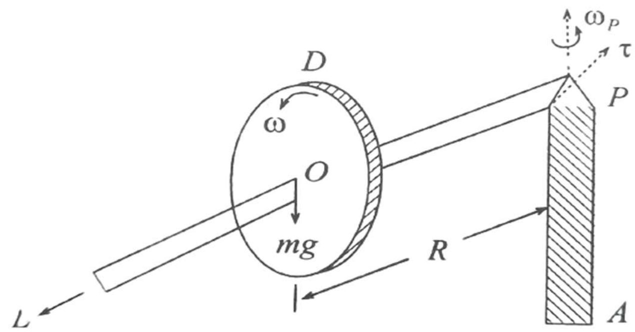
Gyroscope: If the fixed point, about which a symmetrical body is spinning about its axis coincides with the centre of gravity of the body, then it is known as gyroscope.

Principle of working:

1) Gyroscope is a symmetrical body with its axis of spin supported at a point away from its centre of gravity.

2) In figure, 'D' is a heavy disc which is revolving with high angular velocity 'o' about an axle POQ.

3) The axle is supported on a vertical point AP. The weight of the disc 'mg is acting vertically downward at a distance 'R' from P.



4) When the disc is made to spin at a high speed and released with its axis in a horizontal position, a smooth precession takes place around the vertical axis. 5) The gravitational torque acting on the Disc is given by r = mgR where R is the distance between the pivot P and the centre of the Disc.

6) When the disc is not rotating, the gravitational torque simply tends to lower its centre of gravity. But when the disc is rotating about its axis of symmetry, the gravitational torque supplies the torque necessary for precessional motion.

7) In the absence of any external force, the gravitational torque is equal to precessional torque 't'.

In such case $\boldsymbol{\omega_p = \frac{r}{I\omega} = \frac{mgR}{I\omega} = \frac{MgR}{mk^2\omega} = \frac{gR}{k^2\omega}}$
where k is the radius of gyration of the disc.

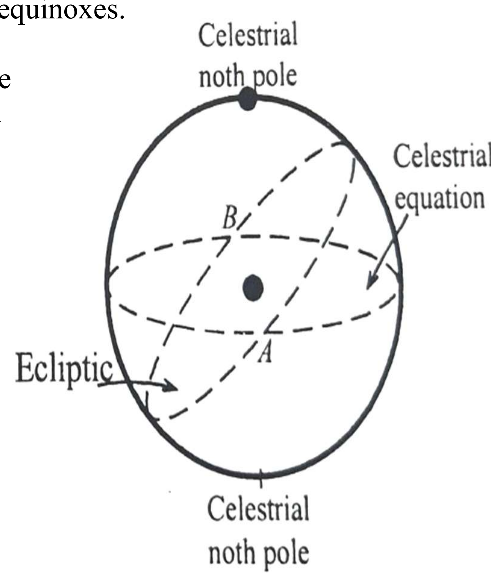
8) If T is the time period of precessional motion, $\boldsymbol{T = \frac{2\pi}{\omega_p} = \frac{\frac{2\pi}{gR}}{k^2\omega} = \frac{2\pi k^2\omega}{gR}}$

This precessional rate is maintained by the gravitational torque.
9) A change in the rate of procession causes the spin axis of the disc to rise or fall. So the spin axis starts oscillating up and down about its mean position of dynamic equilibrium. This type of motion is called nutation.
10) Thus, gyroscopic motion consists of rotation, precession, and nutation.
11) The gyroscope is a device characterised by greater stability of its axis of rotation.
12) This principle is used in the construction of gyrocompass. It accurately indicates the direction of geographic north in ships, submarines, and aeroplanes.

(8) Precession of the Equinoues

A. Precession of equinoxes:

1) The equatorial plane of the earth makes an angle 23-5° with the plane of rotation around sun. The line joining the intersection of these two planes is called the line of equinoxes.



2) During one complete revolution round the sun in one year the earth crosses this line two times once about 21 March and 22nd September.

3) The first point is called equinox and the second one is called the autumnal equinox.

4) The earth is not a perfect sphere. The lower part of the earth is slightly closure than the upper half. This results a difference in the force of attraction.

5) This difference acts as an external torque causing precession of the axis of the rotation of the earth.

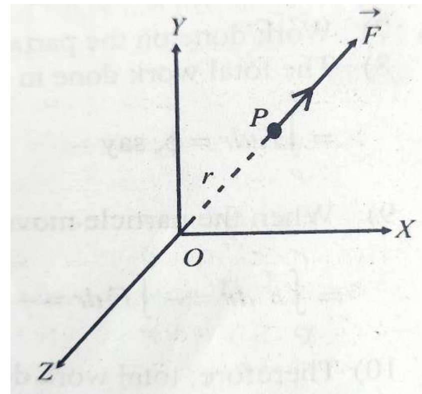
Due to this, the equinoxes precesses.

6) At equinoxes, the duration of day is equal to the duration of night.

UNIT 2

III MOTION IN A CENTRAL FORCE FIELD

(1) Central force Definition & Examples



1) A force which always acts towards or away from a fixed point on a particle or a body is called "central force".

2) The magnitude of such a force depends only on the distance from the fixed point.

Explanation:

1) Let 'O' be the origin of co-ordinate system. P is a particle described by polar co-ordinates r and θ.

2) A force $\vec{F}$ acts on particle P. The central force be represented by $\vec{\boldsymbol{F}} = \boldsymbol{f}(\boldsymbol{r})\hat{\boldsymbol{r}}$ Where $\boldsymbol{f}(\boldsymbol{r})$ is the magnitude of the force which is a function of $\boldsymbol{r}$. $\hat{\boldsymbol{r}}$ is a unit vector along $\vec{\boldsymbol{r}}$ of the particle with reference to the fixed point.

Examples:

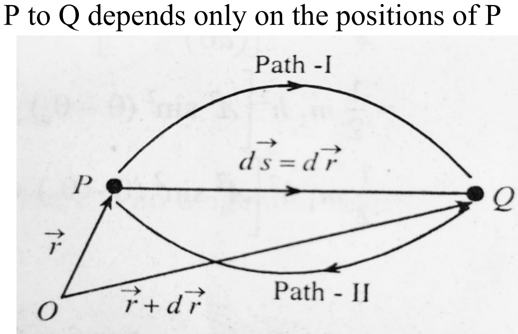
i) The earth moves around the sun under a central force which is always directed towards the sun.
ii) Electrostatic force of attraction or repulsion between two-point charges is a central force.
iii) Elastic force acting on a mass attached to one end of a spring is a central force.

(2) Characteristics of Centra Forces

- The workdone by a central force in a closed path is zero.
- The workdone by a central force in closed path depends only on the positions of body.
- The amount of workdone by the force is same for all paths when the force is conservative.
- The workdone by a central force is independent of the path followed by it.
- Conservative force is a function of position vector of the particle. i.e., $\vec{\boldsymbol{F}} = \boldsymbol{f}(\boldsymbol{r})\hat{\boldsymbol{r}}$.

(3) Conservative nature of central forces

Conservative force:
1) A force is said to be conservative if the work done by it in moving a particle from one point to another in closed path is zero.
2) F is conservative if work done by it in displacing a body from P to Q depends only on the positions of P and Q and not on the path followed by the body.



3) The amount of work done by the force will be the same for all paths when the force is conservative.
4) The work done by a conservative force along a closed path is zero.
5) Let $\vec{\boldsymbol{F}}$ be a force which is a function of position vector $\vec{\boldsymbol{r}}$ of the particle only.

6) When the particle is displaced from the position vector $\vec{\boldsymbol{r}}$ to $\vec{\boldsymbol{r}} + dr$. Then. $ds = dr$

7) Work done on the particle in the displacement is $dw = F - ds = F\,dr = F - dr \rightarrow (1)$

8) The total work done in displacing the particle from P to Q along path-$I = F.dr = W$, say

9) When the particle moves from Q to P, work done along Path-II = W way P

10) Therefore, the total work done along the closed path P→ Q→ P is $W + (-W) = 0$ Thus the work done by a conservation force along a closed path is zero.

(4) Equation of motion under a central force.

Central force: a force which always acts towards or away from a fixed point on a particle or a body is called "central force" the magnitude of such a force depends only on the distance from the fixed point.

Equation of motion of a particle under central forces:

Equation of motion of a particle under central forces:

1) When a body moves under the action of a central force, the force is radial and it is always towards a fixed point.

The radial acceleration $\frac{d^2r}{dt^2} - r(\frac{d\theta}{dt})^2 \rightarrow (1)$

2) Since the force is towards the origin. Since there is no tangential force.

Tangential acceleration = $\frac{1}{r}\frac{d}{dt}\left[r^2\frac{d\theta}{dt}\right] = 0$

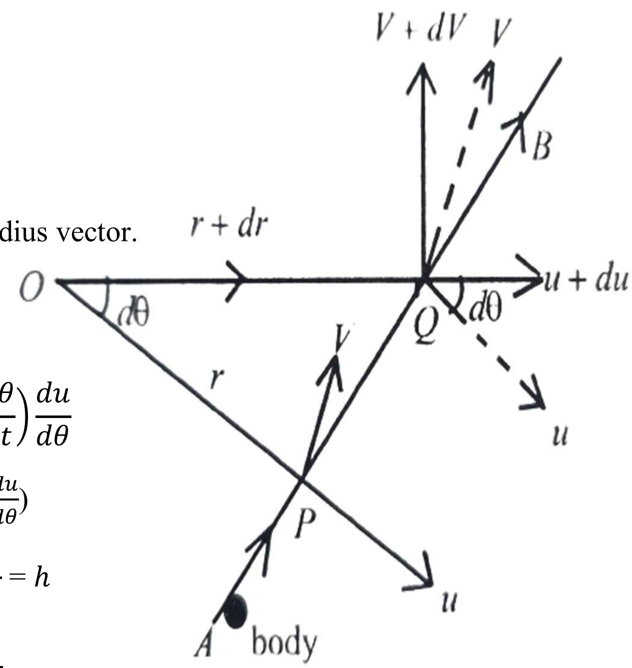
3) r tending to infinity is impossible because in such a case the central force

becomes zero.

Thus, r≠∝ and hence$\frac{1}{r} \neq 0$

4) $r^2\frac{d\theta}{dt} = constant = h\,(say) \rightarrow (2)$

5) Put $u = \frac{1}{r}$ where u is also a variable called reciprocal radius vector.



$$r = \frac{1}{u} \Rightarrow \frac{dr}{dt} = \frac{d}{dt}\left(\frac{1}{u}\right) = -\frac{1}{u^2}\left(\frac{du}{dt}\right) = -\frac{1}{u^2}\left(\frac{du}{d\theta}\right)\left(\frac{d\theta}{dt}\right)$$

$$\frac{dr}{dt} = -\left(r^2\frac{d\theta}{dt}\right)\frac{du}{d\theta}$$

$$\frac{dr}{dt} = -h(\frac{du}{d\theta})$$

Where $r^2\frac{d\theta}{dt} = h$

$$\frac{d\theta}{dt} = \frac{h}{r^2}$$

6) Further

$$\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{dr}{dt}\right) = \frac{d}{dt}\left[-h\frac{du}{d\theta}\right]$$

$$= -h\frac{d^2u}{d\theta^2}.\frac{d\theta}{dt} = -h\frac{d^2u}{d\theta^2}\left(\frac{h}{r^2}\right)$$

$\frac{d^2r}{dt^2} = -h^2u^2\frac{d^2u}{d\theta^2}$ $\quad (\because \frac{1}{r} = u)$

7) Substituting the value of $\frac{d^2r}{dt^2}$ in eq (1)

Radial acceleration $== -h^2u^2\frac{d^2u}{d\theta^2} - r\left(\frac{h^2}{r^4}\right)$

$-h^2u^2\frac{d^2u}{d\theta^2} - \frac{h^2}{r^3} = -h^2u^2\frac{d^2u}{d\theta^2} - h^3u^3$ $(\because \frac{1}{r} = u)$

8) central force acting on the particle is given by $F = Mass$ of a particle × radial acceleration

$$m\left[-h^2u^2\frac{d^2u}{d\theta^2} - h^3u^3\right] = -m\left[h^2u^2\frac{d^2u}{d\theta^2} + h^3u^3\right]$$

Negative sign says that this force is attractive.

$$\frac{F}{m} = \left[h^2u^2\frac{d^2u}{d\theta^2} + h^3u^3\right] = p\ (say)$$

Here p is called central force per unit mass.

$$h^2u^2\left[\frac{d^2u}{d\theta^2} + u\right] = p$$

$$\therefore \frac{d^2u}{d\theta^2} + u = \frac{p}{h^2u^2}$$

This is the differential equation of the orbit of a particle moving under an attractive central force per unit mass (P).

(5) Kepler's laws of planetary motion proofs.

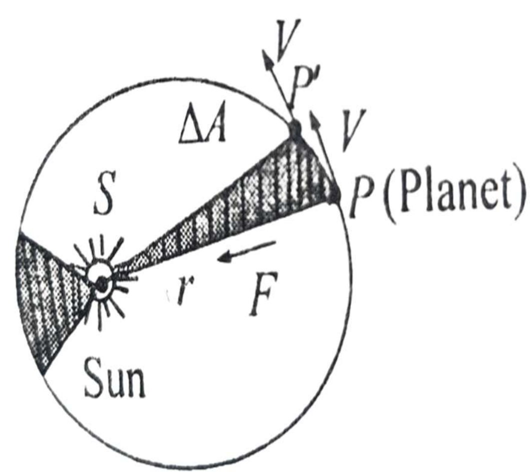
State Kelpler's laws of Planetary motion. Explain of a satellite based on Kepler's laws. Kepler's laws:

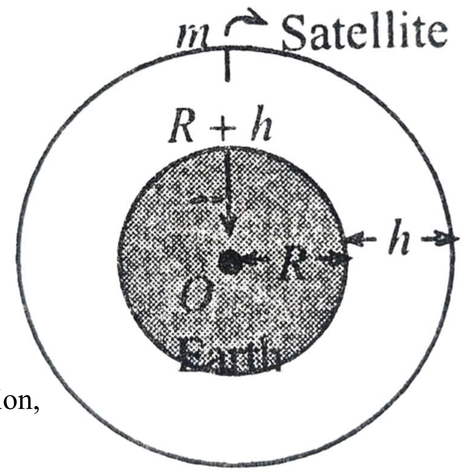
1) I law (or) law of orbits: "All planets move in elliptical orbits with the sun situated at one of the foci".

2) II law (or) law of areas: "The line that joins any planet to the sun sweeps equal areas in equal intervals of time."

3) III law (or) law of periods: "The square of the time period of revolution of a planet is proportional to the cube. of the semi-major axis of the ellipse traced out by the planet

i.e., $T^2 \propto a^3$





Orbital velocity: The velocity required to put the satellite (object) into its orbit around the earth is called orbital velocity (V_0).

Expression:

Let M= mass of the earth, R = radius of the earth,

m = mass of the satellite, V_0 = orbital velocity of satellite,

h = height of the satellite above earth's surface.

$R+h$= orbital radius of the satellite. According to the law of gravitation,

the force of gravity on the satellite is $F = \frac{GMm}{(R+h)^2}$

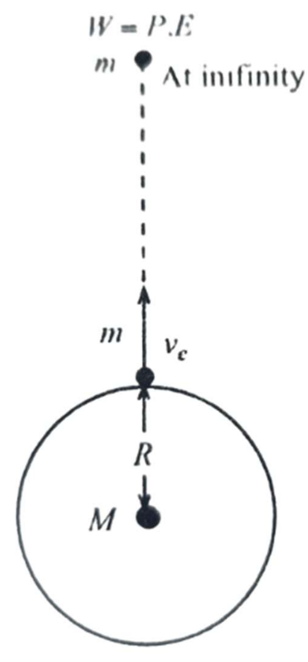
The centripetal force required by the satellite to keep it in its orbit is $F = \frac{mV_0}{(R+h)}$

In equilibrium, the centripetal force is just provided by the gravitational pull of the earth,

So $\frac{mV_0^2}{(R+h)} = \frac{GMm}{(R+h)^2}$ $or V_0^2 = \frac{GM}{(R+h)}$

.. orbital velocity, $V_0 = \sqrt{GM/(R+h)} = \sqrt{gR^2/R+h}$

When the satellite revolves close to the surface of the earth $R+h \sim R. V_0 = \sqrt{gR}.$



Escape velocity: The minimum velocity required for an object to escape- from the gravitational influence of the earth (planet) is known as escape velocity (V_e)

Expression: Consider an object of mass 'm' at rest on the surface of a planet of mass 'M' and radius 'R'.

The force of attraction between the planet and an object is $F = \frac{GMm}{R^2}$

Suppose the object is moving from the surface of the planet to infinity, then the amount of work done against gravitational force is $W = \frac{GMm}{R}$

$$\left[W = F \times R = \frac{GMm}{R^2} \times R\right]$$

This work done will be stored in the object as P.E. For the object to escape up to infinity, the body has to be projected

with a K.E ($\frac{1}{2}mv_e^2$), equal to this gravitational P.E $\left(\frac{GMm}{R}\right)$

Escape velocity $v_e = \sqrt{2gR}$

The escape velocity of an object from the earth's surface is 11-2 km/s.

(6) GPS

Global Positioning System (GPS):

1) It is a system, designed to help navigate on the earth, in the air, and on water.

2) It is made up of satellites, ground stations and receivers.

3) The GPS receiver needs 4 satellites, to work out your position in 3-Dimensions.

4) The GPS is owned and operated by the US department of defence but it is available for general use around the world.

5) 21 GPS satellites and 3 spare satellites are in orbit at 10,600 miles above the earth. Four satellites will be above the horizon.

6) Each satellite contains a computer, an atomic clock, and a radio with an understanding of its own orbit and the clock, it continuously broadcasts is changing position and time.

7) GPS receiver contains a computer that "triangulates" its own position by getting bearings from three of the four satellites, the result is provided in the form of a geographic position longitude and latitude, for most receives within 100 meters.

8) The GPS is being used in science to provide data that has never been available before in the quantity and degree of accuracy that the GPS makes possible.

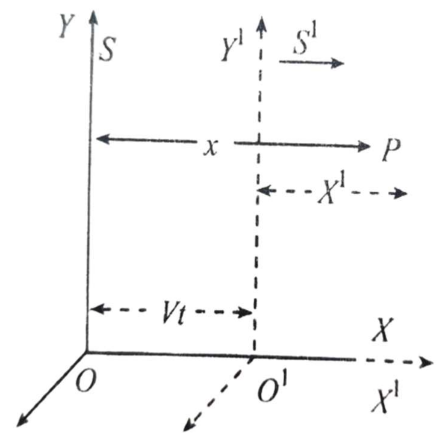
9) The scientists are using the GPS to measure the movement of the arctic ice sheets, the earth's tectonic plates and volcanic activity.

10) A GPS receiver shows where it is. It may also show how fast it is moving, which direction it is going, how high it is, and may be how fast it is going up or down. Many GPS receivers have information about places. The majority are in smart phones.

UNIT 3

IV RELATIVISTIC MECHANICS

(1) Galilean transformations



1) Consider the two frames S and S'. Let the velocity of S' relative to S' be V.

2) Consider an event happening at P at any time.

3) Let the coordinates of P with respect to S be x, y, z, t and with respect to S' be x', y', z', t'.

4) Let X and X' be parallel to v. Let y' and Z' be parallel to Y and Z respectively.

5) Let us also count the time from the instant at which the origin is O and O' coincide.

6) From fig, we have $x = x' + vt$

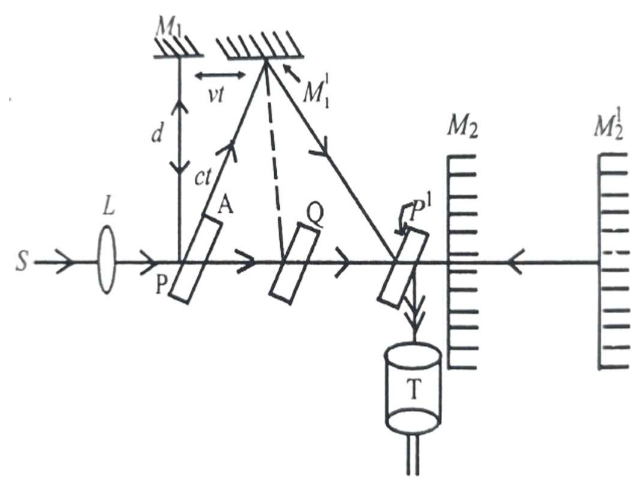
$\therefore x' = x - vt \rightarrow (1)$

7) As there is no relative motion along y and z axis. We have $y' = y$ →(2) and $z' = z \rightarrow (3)$ and $t' = t \rightarrow (4)$

8) Equations (1), (2), (3) and (4) are called Galilean transformations.

(2) Michelson – Morley Experiment

Experiment:



1) The apparatus consists of a monochromatic source of light S.
2) Parallel rays from it are incident on a parallel sided plane glass plate P.
3) They are partly reflected to M_1, and partly transmitted to M_2.
4) The reflected and transmitted parts of the beam are reflected by the plane mirrors M_1, and M_2 and send back to P where they produce an interference pattern which is observed through a telescope T.
5) The paths of the two beams are made exactly equal by introducing another identical glass plate Q parallel to P.
6) Let d = length of optical path of each beam.
c = velocity of light in stationary ether.
v = orbital velocity of ether which is also the velocity of the apparatus.
7) Velocity of light from P to M_2 relative to the apparatus is $c - v$ and on return from M_2 *to* P, it is $c + v$.

8) If t_1, is the time taken by light to travel from P to M_2, and M_2 to P, then

$$t_1 = \frac{d}{c-v} + \frac{d}{c+v} = \frac{2dc}{c^2 - v^2} = \frac{2d}{c(1 - v^2/c^2)} = \frac{2d}{c}\left(1 - \frac{v^2}{c^2}\right) = \frac{2d}{c}(1 + \frac{v^2}{c^2})$$

Using binomial expansion since $v << c$.

9) The light in going from P to M_1, and back, the actual path of light is shown by lines PM_1^1, and M_1^1P' because of the motion of the apparatus through ether.

10) From Δ^{le} $PM_1, M_1^1, c^2t^2 = v^2t^2 + d^2 \Rightarrow d^2 = t^2(c^2 - v^2)$

$\Rightarrow t = \frac{d}{\sqrt{c^2 - v^2}}$

11) If t is the time taken by light to travel from P to M_1^1, then time to travel path $PM_1^1P^1 = t_2 = 2t$.

$$\therefore t = 2t = \frac{2d}{\sqrt{c^2-v^2}} = \frac{2d}{c\sqrt{1-v^2/c^2}} = \frac{2d}{c}(1 - \frac{v^2}{c^2})^{-\frac{1}{2}} = \frac{2d}{c}(1 + \frac{v^2}{2c^2})$$

12) The time difference,

$$\Delta t = t_1 - t_2 = \frac{2d}{c}\{(1 + v/c^2) - (1 + v^2/2c^2)\} = \frac{2d}{c}(v^2/2c^2) = \frac{dv^2}{c^3}[\because t_1 > t_2]$$

13). Optical path difference of the two rays= $\frac{dv^2}{c^3} \times c = \frac{dv^2}{c^2}$

[$\because$optical path difference = Velocity > Δt]

14) If λ is the wavelength of light used, path difference in terms of wavelength $= dv^2/c^2\lambda$.

15) The apparatus was suddenly rotated through 90° the positions of M_1 and M_2 are inter changed. Now the path PM_1, and PM_2, are interchanged. Now the path difference is in opposite directions.

i.e., the path difference is $dv^2/c^2\lambda$. wavelength

16) The resultant path difference = $(-\frac{dv^2}{c^2\lambda} - (\frac{dv^2}{c^2\lambda}) = dv^2/c^2\lambda$ wavelength.

17) Thus, the number of fringes shifted due to rotation of the apparatus $n = 2dv^2/c^2\lambda$

18) In the apparatus used d= 10m, v = 3 x 10 m/sec, λ= 5000 Å, c = 3 x 10^8 m/s.

$\therefore$ n$\frac{2\times10\times(3\times10^4)^2}{(3\times10^8)^2\times5\times10^{-7}} = \frac{2\times9\times10^9}{9\times6\times10^9} = \frac{2}{5} = 0.4$ fringe

Which is almost 0-4 of a fringe width.

19) Thus, a shift of less than half (0.4 fringe) a fringe was only expected. Michel- son and Morley could observe a shift of about 0.01 of fringe. But they could not defect any shift in different seasons on the earth surface. So, it was a null or negative result.

20) This negative or null result suggests that it is impossible to measure the speed of the earth relative to the ether. These observations led Einstein to propose theory of relativity.

(3) Lorentz transformation & Special theory of relativity

Einstein propounded the postulates of special theory of relativity in 1905

They are

1) The laws of physics are the same in all inertial frames of reference.

2) The velocity of light in free space is constant. It is independent of the relative

motion of the source and the observer.

Lorentz transformation equations:

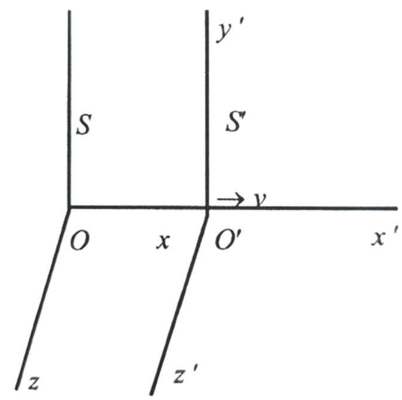
1) S and S' are two frames of reference, S' moving with a uniform velocity v in the $+veX$ - direction relative to frame S. An event occurs in S at a point P whose co-ordinates are (x, y, z, t).

2) The same event is observed in the frame S' and the co-ordinates are (x', y', z', t').

3) The times t and t' are not necessarily the same, though the clocks used in the two frames are perfectly identical and are synchronised at the instant when they pass each other at the origin of co-ordinates. i.e. when O and O' coincide.

4) This means space and time are not separate entities but they are inter related, forming a space time.

5) Let a light source situated at 'O' emit a wave at t=t^1=0 when both the origins of co-ordi-nates O and O' coincide.
6) In frame - S given by



$x^2 + y^2 + z^2 = c^2t^2 \rightarrow (1)$

And in the frame S', the wave front is given by

$x'^2 + y'^2 + z'^2 = c^2t'^2 \rightarrow (2)$

7) From equations (1) and (2), we have

$x^2 + y^2 + z^2 = c^2t^2 = x'^2 + y'^2 + z'^2 = c^2t'^2$

But $y = y'$ and $z = z'$

$\Rightarrow x^2 - c^2t^2 = x'^2 - c'^2t'^2 \rightarrow (3)$

8) Thus, the transformation between x and x', is explained by the simple relationship $x' = K(x - vt) \rightarrow (4)$ where K being independent of x and t.

9) If the system S. is moving relative to S' with a velocity v along '+ ve' x-direction,
then $x = K(x' + vt') \rightarrow (5)$
$\Rightarrow \frac{x}{K} = K(x - vt) + vt' \Rightarrow \frac{x}{K^2} = (x - vt) + \frac{v}{K}t'$

$t' = \frac{K}{v}[\frac{x}{K^2} - (x - vt)]$ $\therefore t' = \frac{K}{v}[x\left(\frac{1}{K^2} - 1\right) + vt] \rightarrow (6)$

10) Substituting (4) and (6) in (3), we get

x^2-$c^2t^2 = [K(x-vt)]^2 - c^2\{K\frac{x}{v}[(\frac{1}{K^2}-1)$+$t^2$ $+\frac{2x}{v}(\frac{1}{K^2}-1)t]\}$

11) Comparing the coefficients off on both sides, we get - $c^2 \ = \ K^2v^2 - c^2K^2$

=>$c^2 \ = \ K^2(c^2-v^2)$ =>$K^2 = \frac{c^2}{c^2-v^2} \rightarrow (7) \therefore K = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \rightarrow (8)$

12) substituting (7) in (6). $t' = \frac{K}{v}[x\left(\frac{c^2-v^2}{c^2}-1\right)+vt] = \frac{K}{v}[-\frac{v^2}{c^2}x+vt]$

$\therefore t' = \frac{1}{\left(1-\frac{v^2}{c^2}\right)}[t-\frac{vx}{c^2}] \rightarrow (9)$ $[\because (8)]$

13) substituting (8) in (4), we get $x' = \frac{1}{1-\frac{v^2}{c^2}}(x-vt) \rightarrow (10)$ and $y' = y \rightarrow (11); z \rightarrow (12)$

14) Equations (9),(10),(11),(12) are called Lorentz transformation equation.

15) when $v \ll c, \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1$, then $x' = x - vt, y' = y, z^2$=z, and $t' = t$.

$[\because$ (9), (10), (11) and (12)] thus for low values of v , Lorentz transformation approach to Galilean transformation.

(4) Time dilation

Time Dilation:

1) Apparent lengthening of time in the moving frame for an observer in the stationary frame is called time dilation.

2) Let *S* and *S′* be the two systems. *S′* moving with velocity v with respect to *S*.

3) Two identical clocks are carried by two systems *S* and *S′* and let both clocks show zero time when their origins *O* and *O′* just cross each other.

4) The time interval between two events as noted by the clock in the moving system (S') is $\Delta t' = t_2' - t_1'$ (1)

5) The time interval between the same two events as noted by the clock in the stationary system

S is $\Delta\, t_2 - t_1$ (2)

6) The Lorentz transformation for t_1 and t_1 are t_1'= $t_1' = \frac{t_1 - vx/c^2}{\sqrt{1-v^2/c^2}}$, $t_2' = \frac{t_2 - vx/c^2}{\sqrt{1-v^2/c^2}}$

7) Substituting the above values in eq (2)

$\Delta t' - t_2' - t_1' = \frac{t_2 - vx/c^2}{\sqrt{1-v^2/c^2}}$ - $\frac{t_1 - vx/c^2}{\sqrt{1-v^2/c^2}}$ $\therefore \Delta t' = \frac{t_2 - t_1}{\sqrt{1-v^2/c^2}} = \frac{\Delta t}{\sqrt{1-v^2/c^2}}$

8) Since $1/\sqrt{1-v^2/c^2}$ is greater than 1, $\Delta t' > \Delta t$.

Thus $\Delta t'$ is greater than Δt i.e., the time interval as observed in S' is greater than that observed in S. This is called time dilation. Thus, clocks go slower in moving frame S' than clock at rest in S frame.

(5) Length Contraction

1) A rod of length / travelling with a very high velocity in a direction parallel to its length appears shorter to a stationary observer. This is called length contraction.

2) Consider two co-ordinate systems S and S'. S' is moving with a velocity V relative to S' along the positive direction of X-axis. Let the rod be at rest in frame S' at rest.

3) Let X_1' and X_2' be the co-ordinates of the ends of the rod of length to in S', at any instant of time.

Then $l_o = X_2' - X_1'$.

4) Let X_1 and X_2 be the coordinates of the rod of length in S. Then $l = X_2 - X_1$.

5) From Lorentz transformations $X_2' = \frac{X_2 - vt}{\sqrt{1-v^2/c^2}}$, $X_1' = \frac{X_1 - vt}{\sqrt{1-v^2/c^2}}$

6) Subtracting $X_2' - X_1' = \frac{X_2 - X_1}{\sqrt{1-v^2/c^2}}$ $\therefore l_0 = l/\sqrt{1-v^2/c^2}$ $\therefore$ $l = l_0\sqrt{1-v^2/c^2}$

Thus $l < l_0$, which means that the length of the rod is contracted by the factor $\frac{1}{\sqrt{1-v^2/c^2}}$ as measured by an observer in S

as measured by an observer in S.

(6) E=MC^2

According to Newtons second law of motion

Force = rate of change of momentum.

$$F = \frac{d}{dt}(mv) = m\,\frac{dv}{dt} + v\left(\frac{dm}{dt}\right)dx$$

Since changes both in mass and velocity must be considered in relativistic mechanics.

2) work done by this force in displacing the body through a distance dx is dw

$= F.dx = m\left(\frac{dv}{dt}\right).dx + v\,dm/(dt\,dx)$

$= mdv\left(\frac{dx}{dt}\right) + v\,dm\,(dx/dt)$ [since $\frac{dx}{dt} = v$]

$= mv\,dv = v^2\,dm \ldots\ldots(1)$

Where V is the velocity of the particle.

3) But according to the relativity theory $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$ i.e, $m^2c^2 - m^2v^2 = m_0^2\,c^2$

4) Differentiating this equation $c^2 2m\,dm - m^2\,.\,2v\,dv - v^2.\,2m.dm = 0$

$c^2dm = mv\,dv + v^2\,dm \ldots.(2)$

5) Comparing (1) and (2), $dw = c^2\ dm$

This is the kinetic energy of the body.

$dE = \ c^2 . dm$

6) when the body accelerated from rest to a velocity v mass change m_o to m

Then $E = \int dE = \ \int_{m_o}^{m} c^2\ dm = mc^2 - m_o\ c^2 = mc^2$.

7) but rest mass energy = $m_o c^2$.

8) the total energy = $mc^2 - m_o\ c^2 = mc^2$

UNIT 4

UNDAMPED, DAMPED AND FORCED OSCILLATIONS

(1) Forced harmonic oscillator

1) Forced oscillations Besides damping if an external periodic force i acting on the body、 then the oscillations are called forced oscillations

2) Differential equation of motion of a damped oscillator will be as below $m\frac{d^2y}{dt^2}+r\frac{dy}{dt}+ay=0$

3) Now suppose an extertal periodic force $f = 1\, sin\, \omega\, t$ is acting on the body Then the equation of motion can be written as below

$m\frac{d^2y}{dt^2}+r\frac{dy}{dt}+ay=fsin\,\omega\, t$

were $\frac{r}{m}=2b, \frac{a}{m}=\omega_0^2 \& \frac{f_0}{m}=F_0$

4)The solution of this equation will be as below $y = Asin \quad (\omega\, t + \emptyset)$

A is the amplitude and $\emptyset$ initial phase. The resultant displacement y varic with the frequency of the periodic force o and not with the natural frequency of the body ω_0.

5) From the above, $\frac{dy}{dt}=A\,\omega\, cos(\omega t+\emptyset)$

6) Substituting these values in eq. (1 $Awsin(\omega t+\emptyset)+2bA\,\omega\, cos(\omega t+\emptyset)+wAsin(\omega t+\emptyset)wt+\emptyset)= Fsin\,\omega t$

(2) Resonance

Resonance: If a vibrating body vibrates due to forced vibrations at another body, [AU A19; SVU A18] and if the frequency of vibrating body is equal to the forced vibrations, then the two vibrations are said to be in resonance.

E.g. 1. Vibrating particle in an air column vibrates due to vibrating turning fork. If the frequency of turning fork is equal to the frequency of air column, resonance takes place. The we can near forced sound.

E.g. 2. Marching soldiers are asked to dispose while crossing the bridge, because if the frequency of the steps of the marching soldiers is equal to the frequency of the bridge vibrations, then the bridge collapses due to resonance.

(3) Logarithmic decrement

1) The logarithmic decrement of a damped oscillator indicates the rate at which the amplitude of the oscillator decreases.

2) The amplitude of the damped simple harmonic motion is given by $a = A\,e$

3) When t=0 the amplitude $a = Ap$. Let the amplitudes be a, a2 a3 at instances when $t = 2T/2, 3T/2,$ respectively. Where T is the time period of the oscillator.

4) Then $a_,$= Ag e a(T/2) 42 = A (21/2)

5) From the above equations

6) The constant d is called decrement. It denotes the ratio of two successive amplitudes. The natural logarithm of this decrement is called logarithmic decrement (A) of the oscillator.

7) The natural logarithm of the ratio of successive amplitudes during the time interval of $T/2$ is called logarithmic decrement of the oscillator.

(3) Relaxation time

1) The time taken for the total energy to decay to (1/e) of its original value is defined as relaxation time.

2) The energy of damped harmonic oscillator

3) When t=0 then E= E, E= E e2b → (1)

4) Let T be the relaxation time. E=E/e 5) Then eq (1) becomes i.e., when t=T E Ene 2 2 1 = 2br 6) Also if power dissipation is P then =P/E and if is the quality factor then=2 where o is the angular frequency.

(4) Quality factor

1.Quality factor of an oscillator is defined as the ratio of the ratio of the energy of the oscillator to the energy lost per radian of the angular frequency.

$$\text{i.e, } Q = \frac{\textbf{Energy of oscillator}}{\textbf{Energy lost per cycle}/2\pi} = 2\pi \frac{\textbf{Energy of oscillator}}{\textbf{Energy lost per cycle}}$$

$$Q = \frac{\omega}{\omega_2 - \omega_1} = \omega r$$

2.We know that the energy of an undamped simple harmonic oscillator = $\frac{1}{2} mA^2 \omega_0^2$ where m is its mass, A is its amplitude and ω_0 is its natural angular frequency.

3.But in a damped oscillator work is done against resistive forces. So total energy of the oscillator goes on decreasing.

4.Amplitude of the damped oscillator = $A = A_0 e^{-bt}$, Energy of the damped oscillator

$$= \mathbf{E} = \frac{1}{2} mA_0^2 e^{-2bt} \omega'^2$$

Since the angular frequency changes to ω'

5.power dissipated per cycle = P = Rate of loss of energy with time.

$$-\frac{dE}{dt} = -m\omega'^2 A_0 e^{-bt}(-2b. = bm)\omega'^2 A_0^2 e^{-2bt} = 2b.(E)$$

6.Power dissipation = $P = 2bET$ Where T is the period of damped oscillation.

$$Q = \frac{2\pi \cdot E}{2bET} = \pi/bT.$$

7. But $T = \frac{2\pi}{\omega_0^2 - b^2}$

$$Q\frac{\pi\sqrt{\omega_0^2 - b^2}}{b \cdot 2\pi} = \frac{\sqrt{\omega_0^2 - b^2}}{2b} = \omega' 2b$$

Where $\omega' = \sqrt{\omega_0^2 - b^2}$

8.thus the quality factor will be large if the damping coefficient K is small thus it represents the efficiency of the oscillator

UNIT V

VIBRATING STRINGS

1) Transverse wave propagation along a stretched string

Transverse waves: Waves produced when the particles of the medium are vibrating perpendicular to the direction of propagation of waves are called transverse waves.

Velocity of transverse waves in strings

1) A string AB stretched under tension T is plucked at the middle point and released. T MS

2) When it begins to vibrate, the component of the tension at right angles to the string tends PA e to bring the string back to its original position. 0-80 RMA &

3) PO is a small portion of the string

4) The tension at P acts along the tangent PR while that at acts along the tangent QS.

5) The tangents PR and QS are inclined to the x-axis at an angle 0 and 0- 80 respectively.

6) The angle 60 is very small as the amplitude of the string is very small.

7) Component of tension T acting at P in the horizontal direction is T cos 0 and in vertical direction it is T sin

8) Component of tension T acting at Q in the horizontal direction T cos (0- 80) and in vertical direction it is T sin (0-80)

9) T cos 0 and Tcos (0-80) are nearly equal. Since 80 is very small they cancel each other as they act in opposite directions

10) Resultant force in vertically downward direction = $Tsin\ 0\ -\ T\ sin\ (0-\ \delta 0)$

$$F + T\{sin\theta - \sin(\theta - \delta\theta)\}$$

$$= T\{sin\theta - sin\theta cos\delta\theta + \cos sin\delta\theta\} = T\ Cos\theta\delta\theta$$

$$[sin\delta\theta = \delta\theta\ and \cos\delta\theta = 1$$

$$F = Td(sin\theta)$$

$$F = T\frac{d}{dx}\left(\frac{dy}{dx}\right)\delta x = T\frac{d^2y}{dx^2}\delta x\ \rightarrow (1)$$

11) If m is the mass per unit length of the wire, the force on the element PQ of the wire = $m.\delta x\ \frac{d^2y}{dt^2}$(2) dt

12) From (1) and (2) we get $m\delta\frac{d^2y}{dt^2} = T\frac{d^2y}{dx^2}.\delta x$

$$\frac{d^2y}{dt^2} = \frac{T}{m}\frac{d^2y}{dx^2}$$

13) Comparing this with the differential equation of wave motion

$\frac{d^2y}{dt^2} = V^2 \frac{d^2y}{dx^2}$ we get ,V^2=$\frac{T}{m}$ velocity V=$\sqrt{T/m} \rightarrow 4$

Laws of Transverse Vibrations:

I law: At a constant tension and constant linear density the frequency of the fundamental note emitted by a string is inversely proportional to its length i.e., $v\ o\ 1/l\ when\ T\ and\ m'$ are constants.

II law: When the length and linear density are constant the fundamental note emitted is directly proportional to the square root of Tension $v\ o\ VT$ when I and m are constants.

III law: When the length and tension are constant the fundamental note emitted is inversely proportional to the square root of mass per unit length, $u, v\ o\ 1/Nm$ when land T are constant.

2) General solution of wave equation

1) Let any arbitrary function of either $(x - Vt)$ or $(x + Vt)$ be the solution for the equation $\left(\frac{d^2y}{dt^2} = V^2 \frac{d^2y}{dx^2}\right)$

2) then the most general solution will be $Y = f(x \pm Vt) \rightarrow (1)$

3) Differentiating eq (1) w.r.t to x

$$\frac{dy}{dx} = f'(x \pm Vt) and\ \frac{d^2y}{dx^2} = f''\ (x \pm Vt) \rightarrow (2)$$

4) Differentiating eq (1) w.r.t to $'t'$

$$\frac{dy}{dx} = Vf'(x \pm Vt)$$

$$\frac{d^2y}{dt^2} = V^2 f''(x \pm Vt) \rightarrow (3)$$

5) from eq (2) and eq (3)

$$\frac{d^2y}{dt^2} = V^2 \frac{d^2y}{dx^2} \rightarrow (4)$$

Eq. (4) represents differential form of wave equation.

3) Modes of vibration of stretched string damped of both the ends

A. Consider the most general simple harmonic solution of the wave equation in case of a uniform string of length / having mass per unit length m and stretched by a tension T.

2) The general solution of the wave equation is given by $Y = a, sin(ot - kx) + a, sin\ (ot - kx) + b, cos\ (ot - kx) - + b, cos\ (t - + kx) -> (1)$ where $a, 'a2, b, and\ b,$ are arbitary constants

3) As the string is rigidly supported at the two ends, we have following boundary conditions. $Y = 0\ at\ x = 0\ at\ all\ time\ t -> (2)\ Y = 0\ at\ x = l\ at\ all\ time\ t -> (3)$

4) Applying boundary condition (2) in eq (1) $O = a_1 \ sin\ \omega t + a \sin \omega t + b_1 \cos \omega t + b_2\ cos\ \omega t$

$$o = (a + a \sin \omega\ t + (b + b) \cos \omega t$$

5) At $\sin \omega t \neq 0\ and \cos \omega t \neq 0$

$Hence\ a_1 + a_2 = 0\ and\ b_1 + b_2 = 0\ \ Thus\ a_1 = -a_2\ and\ b_1 = -b_2$

6)Now eq (1) becomes $Y = a, [\sin(\omega t - kx) - \sin(\omega t - + kx)] + b, (\cos(\omega t - kx) - \cos(t - + kx)]$

$$= altsin\ ot\ cos\ kx - cos\ ot\ sin\ kx) - (sin\ ot\ coskx + cos\ ot\ sin\ kx) + b, [fcos\ ot\ coskx + sin\ ot\ sin\ kx) - fcos\ wt\ cos\ kx - sin\ ot\ sin\ kxy\ 2a, cos\ t\ sin\ kx + 2b, sin\ \omega t\ sin\ kx - Y = (2a, cos\ \omega t + 2b, sin\ \omega t)\ sin\ kx -> (4)$$

ULTRASONICS

1) Ultrasonics

Ultrasonics The acoustic waves having frequencies higher than the audible range (Above 20,000 Hz) are called ultrasonics

2) Production of ultrasonics by magnetostriction method:

Ultrasonics The acoustic waves having frequencies higher than the audible range (Above 20,000 Hz) are called ultrasonics Production of ultrasonic waves: Magneto striction method:

1) "When a ferromagnetic material is magnetized, certain changes occur in its internal structure and the resulting stress in the specimen produces small changes in its physical dimensions. This phenomenon is called magnetostriction"

2) The length of a rod made of a ferromagnetic material increases slightly, when a magnetic field is applied parallel to its length.

3) When a rod made of, say Nickel is subjected to an alternating magnetic field, the length of the rod also varies between normal and elongation due to demagnetisation and magnetisation

4) A Nickel rod is clamped at the centre of a coil carrying steady current. Two coils and B are wound around the rod.

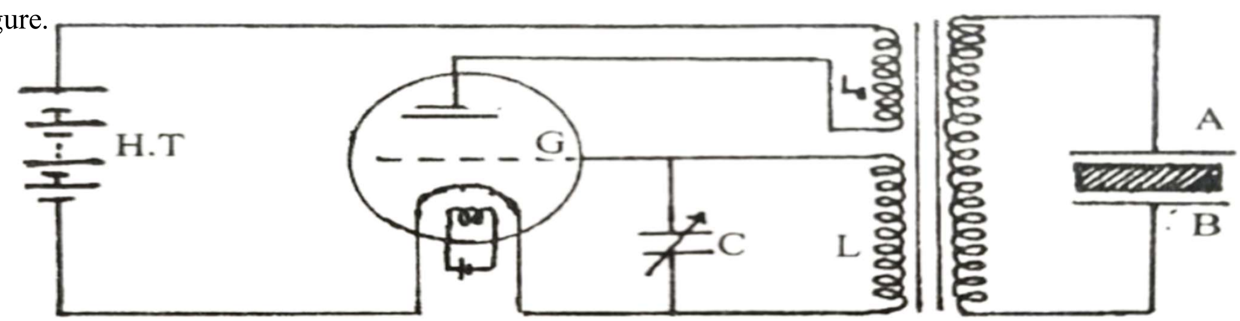
5) A is inserted in the tuned plate circuit of frequency oscillator circuit. is magnetically coupled to A and is connected in the grid circuit. helps to increase the energy of the oscillations, 6) The natural frequency of the rod is given by $n = \frac{l}{2l}\sqrt{\frac{y}{\rho}}$

7) The capacity of the condenser C is charged under the frequency of the oscillations becomes equal to the natural frequency of the rod. Then oscillations are produced in the rod in resonance and given ultrasonics of high frequency.

3) Production of ultrasonic by Piezo electric method.

Piezo electric method: I) "When certain crystals like quartz, tourmaline, rock salt are stretched (or) compressed along a certain axis, an electrical P.D is formed along a perpendicular axis, this is called Piezo electric effect".

2) An A- cut quartz crystal is taken. It is subjected to alternating electric field of high frequency as shown in figure.



3) In figure L- C is the tank circuit kept in the grid circuit. C is a variable capacitor and L, is a coil called tickler coil coupled with coil L.

4) When high tension (H.T) is applied, high frequency oscillating current in the tank circuit induces an e.m.f. of same frequency into the circuit containing X- cut quartz crystal.

5) The frequency of this voltage applied to the crystal can changed by varying C. When the frequency of the tank circuit is equal to the natural frequency of the crystal, resonance occurs and the crystal produce ultrasonic waves.

4) Detection of ultrasonics and applications.

1) Detection of Ultrasonics:

Thermal detectors • This is most used for detecting ultrasonics. In this a platinum wire is moved is moved through the medium. At the positions of nodes, due to alternate Compressions and rarefactions, adiabatic changes in temperature take place. The change in resistance can be detected with a very sensitive Callender and Griffiths bridge. At nodes the temperature remains constant and there will be no change in the balanced bridge.

2) Quartz crystal method:

This method of detection is based on the principle of piezo electric effect. When one pair of the opposite faces of a quartz crystal is exposed to ultrasonic waves, the other pair of opposite faces develop opposite but equal charges. These charges cause a potential difference which can be amplified and detected using electric circuit.

Sensitive flame method: A narrow sensitive flame is moved along the medium, where there are antinodes, the flame is steady. Where there are nodes, there the flames flickers because of the change in pressure. Thus, the positions of nodes and antinodes can be located. The average distance bet ween two nodes gives $\frac{lambda}{2}$ which iS very small if the vibrations are ultrasonic.

4**) Kundt's Tube Method:** This method is useful for detecting ultrasonic waves of low frequency. When ultrasonic waves are passed through Kundt's tube, lycopodium powder sprinkled in the tube collects in the form of heaps at nodes. The average distance between two consecutive heaps gives the value of half the wavelength. This method, however, fails if wavelength of ultrasonics waves is very small i.e., less than in few mm.

5) Properties of Ultrasonics.

1.Ultrasonics are used to investigate the structure of matter.

2) Ultrasonics are used to detect flaws in metal castings.

3) Ultrasonics are used to determine depth of sealakes etc and to detect the submerged rocks, submarines, and ice bergs.

4) Ultrasonics are used to for sterilisation of milk and water.